CHAPTER 1 TEST

1. **(a)** Graph the intervals $(-5, 3]$ and $(2, \infty)$ on the real number line.
   (b) Express the inequalities $x \le 3$ and $-1 \le x < 4$ in interval notation.
   (c) Find the distance between -7 and 9 on the real number line.

2. Evaluate each expression.
   (a) $(-3)^4$ **(b)** -3^4 **(c)** 3^{-4} **(d)** $\dfrac{5^{23}}{5^{21}}$ **(e)** $\left(\dfrac{2}{3}\right)^{-2}$ **(f)** $16^{-3/4}$

3. Write each number in scientific notation.
   (a) 186,000,000,000 **(b)** 0.0000003965

4. Simplify each expression. Write your final answer without negative exponents.
   (a) $\sqrt{200} - \sqrt{32}$ **(b)** $(3a^3b^3)(4ab^2)^2$ **(c)** $\left(\dfrac{3x^{3/2}y^3}{x^2y^{-1/2}}\right)^{-2}$
   (d) $\dfrac{x^2 + 3x + 2}{x^2 - x - 2}$ **(e)** $\dfrac{x^2}{x^2 - 4} - \dfrac{x + 1}{x + 2}$ **(f)** $\dfrac{\dfrac{y}{x} - \dfrac{x}{y}}{\dfrac{1}{y} - \dfrac{1}{x}}$

5. Rationalize the denominator and simplify: $\dfrac{\sqrt{10}}{\sqrt{5} - 2}$

6. Perform the indicated operations and simplify.
   (a) $3(x + 6) + 4(2x - 5)$ **(b)** $(x + 3)(4x - 5)$ **(c)** $(\sqrt{a} + \sqrt{b})(\sqrt{a} - \sqrt{b})$
   (d) $(2x + 3)^2$ **(e)** $(x + 2)^3$

7. Factor each expression completely.
   (a) $4x^2 - 25$ **(b)** $2x^2 + 5x - 12$ **(c)** $x^3 - 3x^2 - 4x + 12$
   (d) $x^4 + 27x$ **(e)** $3x^{3/2} - 9x^{1/2} + 6x^{-1/2}$ **(f)** $x^3y - 4xy$

8. Find all real solutions.
   (a) $x + 5 = 14 - \frac{1}{2}x$ **(b)** $\dfrac{2x}{x + 1} = \dfrac{2x - 1}{x}$ **(c)** $x^2 - x - 12 = 0$
   (d) $2x^2 + 4x + 1 = 0$ **(e)** $\sqrt{3 - \sqrt{x + 5}} = 2$ **(f)** $x^4 - 3x^2 + 2 = 0$
   (g) $3|x - 4| = 10$

9. Perform the indicated operations, and write the result in the form $a + bi$.
   (a) $(3 - 2i) + (4 + 3i)$ **(b)** $(3 - 2i) - (4 + 3i)$
   (c) $(3 - 2i)(4 + 3i)$ **(d)** $\dfrac{3 - 2i}{4 + 3i}$
   (e) i^{48} **(f)** $(\sqrt{2} - \sqrt{-2})(\sqrt{8} + \sqrt{-2})$

10. Find all real and complex solutions of the equation $2x^2 + 4x + 3 = 0$.

11. Mary drove from Amity to Belleville at a speed of 50 mi/h. On the way back, she drove at 60 mi/h. The total trip took $4\frac{2}{5}$ h of driving time. Find the distance between these two cities.

12. A rectangular parcel of land is 70 ft longer than it is wide. Each diagonal between opposite corners is 130 ft. What are the dimensions of the parcel?

13. Solve each inequality. Write the answer using interval notation, and sketch the solution on the real number line.
    (a) $-4 < 5 - 3x \le 17$ **(b)** $x(x - 1)(x + 2) > 0$
    (c) $|x - 4| < 3$ **(d)** $\dfrac{2x - 3}{x + 1} \le 1$

14. A bottle of medicine is to be stored at a temperature between 5°C and 10°C. What range does this correspond to on the Fahrenheit scale? [*Note:* Fahrenheit (F) and Celsius (C) temperatures satisfy the relation $C = \frac{5}{9}(F - 32)$.]

15. For what values of x is the expression $\sqrt{6x - x^2}$ defined as a real number?

16. **(a)** Plot the points $P(0, 3)$, $Q(3, 0)$, and $R(6, 3)$ in the coordinate plane. Where must the point S be located so that $PQRS$ is a square?

(b) Find the area of $PQRS$.

17. **(a)** Sketch the graph of $y = x^2 - 4$.

(b) Find the x- and y-intercepts of the graph.

(c) Is the graph symmetric about the x-axis, the y-axis, or the origin?

18. Let $P(-3,1)$ and $Q(5,6)$ be two points in the coordinate plane.

(a) Plot P and Q in the coordinate plane.

(b) Find the distance between P and Q.

(c) Find the midpoint of the segment PQ.

(d) Find the slope of the line that contains P and Q.

(e) Find the perpendicular bisector of the line that contains P and Q.

(f) Find an equation for the circle for which the segment PQ is a diameter.

19. Find the center and radius of each circle, and sketch its graph.

(a) $x^2 + y^2 = 25$ **(b)** $(x - 2)^2 + (y + 1)^2 = 9$ **(c)** $x^2 + 6x + y^2 - 2y + 6 = 0$

20. Write the linear equation $2x - 3y = 15$ in slope-intercept form, and sketch its graph. What are the slope and y-intercept?

21. Find an equation for the line with the given property.

(a) It passes through the point $(3, -6)$ and is parallel to the line $3x + y - 10 = 0$.

(b) It has x-intercept 6 and y-intercept 4.

22. A geologist measures the temperature T (in °C) of the soil at various depths below the surface and finds that at a depth of x cm, the temperature is given by $T = 0.08x - 4$.

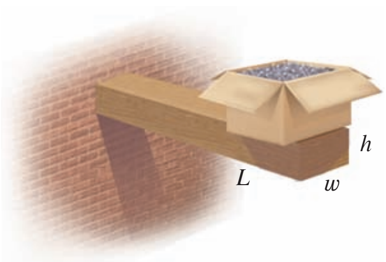
(a) What is the temperature at a depth of 1 m (100 cm)?

(b) Sketch a graph of the linear equation.

(c) What do the slope, the x-intercept, and T-intercept of the graph represent?

23. Solve the equation and the inequality graphically.

(a) $x^3 - 9x - 1 = 0$ **(b)** $x^2 - 1 \le |x + 1|$



24. The maximum weight M that can be supported by a beam is jointly proportional to its width w and the square of its height h and inversely proportional to its length L.

(a) Write an equation that expresses this proportionality.

(b) Determine the constant of proportionality if a beam 4 in. wide, 6 in. high, and 12 ft long can support a weight of 4800 lb.

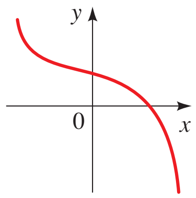
(c) If a 10-ft beam made of the same material is 3 in. wide and 10 in. high, what is the maximum weight it can support?

If you had difficulty with any of these problems, you may wish to review the section of this chapter indicated below.

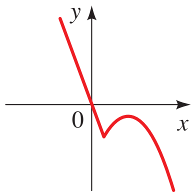
Problem	Section	Problem	Section
1	Section 1.1	13, 14, 15	Section 1.8
2, 3, 4(a), 4(b), 4(c)	Section 1.2	23	Section 1.11
4(d), 4(e), 4(f), 5	Section 1.4	16, 17, 18(a), 18(b)	Section 1.9
6, 7	Section 1.3	18(c), 18(d)	Section 1.10
8	Section 1.5	18(e), 18(f), 19	Section 1.9
9, 10	Section 1.6	20, 21, 22	Section 1.10
11, 12	Section 1.7	24	Section 1.12

1. Which of the following are graphs of functions? If the graph is that of a function, is it one-to-one?

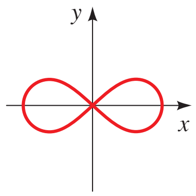
(a)



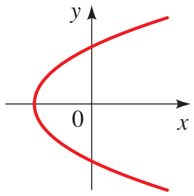
(b)



(c)



(d)



2. Let $f(x) = \dfrac{\sqrt{x}}{x + 1}$.

   (a) Evaluate $f(0)$, $f(2)$, and $f(a + 2)$.

   (b) Find the domain of f.

   (c) What is the average rate of change of f between $x = 2$ and $x = 10$?

3. A function f has the following verbal description: "Subtract 2, then cube the result."

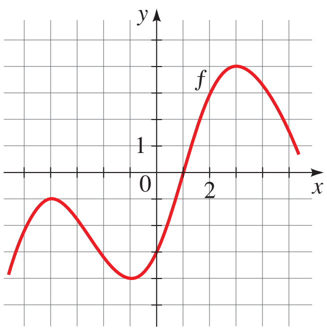
   (a) Find a formula that expresses f algebraically.

   (b) Make a table of values of f, for the inputs -1, 0, 1, 2, 3, and 4.

   (c) Sketch a graph of f, using the table of values from part (b) to help you.

   (d) How do we know that f has an inverse? Give a verbal description for f^{-1}.

   (e) Find a formula that expresses f^{-1} algebraically.



4. A graph of a function f is given in the margin.

   (a) Find the local minimum and maximum values of f and the values of x at which they occur.

   (b) Find the intervals on which f is increasing and on which f is decreasing.

5. A school fund-raising group sells chocolate bars to help finance a swimming pool for their physical education program. The group finds that when they set their price at x dollars per bar (where $0 < x \le 5$), their total sales revenue (in dollars) is given by the function $R(x) = -500x^2 + 3000x$.

   (a) Evaluate $R(2)$ and $R(4)$. What do these values represent?

   (b) Use a graphing calculator to draw a graph of R. What does the graph tell us about what happens to revenue as the price increases from 0 to 5 dollars?

   (c) What is the maximum revenue, and at what price is it achieved?

6. Determine the net change and the average rate of change for the function $f(t) = t^2 - 2t$ between $t = 2$ and $t = 2 + h$.

7. Let $f(x) = (x + 5)^2$ and $g(x) = 1 - 5x$.

   (a) Only one of the two functions f and g is linear. Which one is linear, and why is the other one not linear?

   (b) Sketch a graph of each function.

   (c) What is the rate of change of the linear function?

8. **(a)** Sketch the graph of the function $f(x) = x^3$.

   (b) Use part (a) to graph the function $g(x) = (x - 1)^3 - 2$.

9. **(a)** How is the graph of $y = f(x - 3) + 2$ obtained from the graph of f?

   (b) How is the graph of $y = f(-x)$ obtained from the graph of f?

10. Let $f(x) = \begin{cases} 1 - x & \text{if } x \le 1 \\ 2x + 1 & \text{if } x > 1 \end{cases}$

(a) Evaluate $f(-2)$ and $f(1)$.

(b) Sketch the graph of f.

11. If $f(x) = x^2 + x + 1$ and $g(x) = x - 3$, find the following.

(a) $f + g$ **(b)** $f - g$ **(c)** $f \circ g$ **(d)** $g \circ f$

(e) $f(g(2))$ **(f)** $g(f(2))$ **(g)** $g \circ g \circ g$

12. Determine whether the function is one-to-one.

(a) $f(x) = x^3 + 1$ **(b)** $g(x) = |x + 1|$

13. Use the Inverse Function Property to show that $f(x) = \dfrac{1}{x - 2}$ is the inverse of $g(x) = \dfrac{1}{x} + 2$.

14. Find the inverse function of $f(x) = \dfrac{x - 3}{2x + 5}$.

15. **(a)** If $f(x) = \sqrt{3 - x}$, find the inverse function f^{-1}.

(b) Sketch the graphs of f and f^{-1} on the same coordinate axes.

16–21 ■ A graph of a function f is given below.

16. Find the domain and range of f.

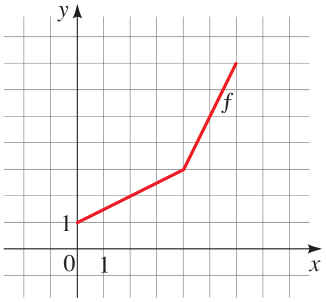
17. Find $f(0)$ and $f(4)$.

18. Graph $f(x - 2)$ and $f(x) + 2$.

19. Find the net change and the average rate of change of f between $x = 2$ and $x = 6$.

20. Find $f^{-1}(1)$ and $f^{-1}(3)$.

21. Sketch the graph of f^{-1}.



22. Let $f(x) = 3x^4 - 14x^2 + 5x - 3$.

(a) Draw the graph of f in an appropriate viewing rectangle.

(b) Is f one-to-one?

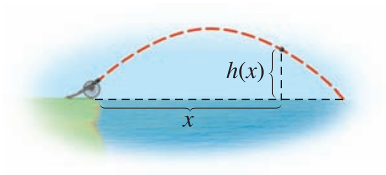
(c) Find the local maximum and minimum values of f and the values of x at which they occur. State each answer correct to two decimal places.

(d) Use the graph to determine the range of f.

(e) Find the intervals on which f is increasing and on which f is decreasing.

1. Express the quadratic function $f(x) = x^2 - x - 6$ in standard form, and sketch its graph.

2. Find the maximum or minimum value of the quadratic function $g(x) = 2x^2 + 6x + 3$.



3. A cannonball fired out to sea from a shore battery follows a parabolic trajectory given by the graph of the equation

$$h(x) = 10x - 0.01x^2$$

where $h(x)$ is the height of the cannonball above the water when it has traveled a horizontal distance of x feet.

(a) What is the maximum height that the cannonball reaches?

(b) How far does the cannonball travel horizontally before splashing into the water?

4. Graph the polynomial $P(x) = -(x + 2)^3 + 27$, showing clearly all x- and y-intercepts.

5. **(a)** Use synthetic division to find the quotient and remainder when $x^4 - 4x^2 + 2x + 5$ is divided by $x - 2$.

(b) Use long division to find the quotient and remainder when $2x^5 + 4x^4 - x^3 - x^2 + 7$ is divided by $2x^2 - 1$.

6. Let $P(x) = 2x^3 - 5x^2 - 4x + 3$.

(a) List all possible rational zeros of P.

(b) Find the complete factorization of P.

(c) Find the zeros of P.

(d) Sketch the graph of P.

7. Find all real and complex zeros of $P(x) = x^3 - x^2 - 4x - 6$.

8. Find the complete factorization of $P(x) = x^4 - 2x^3 + 5x^2 - 8x + 4$.

9. Find a fourth-degree polynomial with integer coefficients that has zeros $3i$ and -1, with -1 a zero of multiplicity 2.

10. Let $P(x) = 2x^4 - 7x^3 + x^2 - 18x + 3$.

(a) Use Descartes' Rule of Signs to determine how many positive and how many negative real zeros P can have.

(b) Show that 4 is an upper bound and -1 is a lower bound for the real zeros of P.

(c) Draw a graph of P, and use it to estimate the real zeros of P, rounded to two decimal places.

(d) Find the coordinates of all local extrema of P, rounded to two decimals.

11. Consider the following rational functions:

$$r(x) = \frac{2x - 1}{x^2 - x - 2} \qquad s(x) = \frac{x^3 + 27}{x^2 + 4} \qquad t(x) = \frac{x^3 - 9x}{x + 2} \qquad u(x) = \frac{x^2 + x - 6}{x^2 - 25} \qquad w(x) = \frac{x^3 + 6x^2 + 9x}{x + 3}$$

(a) Which of these rational functions has a horizontal asymptote?

(b) Which of these functions has a slant asymptote?

(c) Which of these functions has no vertical asymptote?

(d) Which of these functions has a "hole"?

(e) What are the asymptotes of the function $r(x)$?

(f) Graph $y = u(x)$, showing clearly any asymptotes and x- and y-intercepts the function may have.

(g) Use long division to find a polynomial P that has the same end behavior as t. Graph both P and t on the same screen to verify that they have the same end behavior.

12. Solve the rational inequality $x \le \dfrac{6 - x}{2x - 5}$.

13. Find the domain of the function $f(x) = \dfrac{1}{\sqrt{4 - 2x - x^2}}$.

14. (a) Choosing an appropriate viewing rectangle, graph the following function and find all its x-intercepts and local extrema, rounded to two decimals.

$$P(x) = x^4 - 4x^3 + 8x$$

(b) Use your graph from part (a) to solve the inequality

$$x^4 - 4x^3 + 8x \ge 0$$

Express your answer in interval form, with the endpoints rounded to two decimals.

1. Sketch the graph of each function, and state its domain, range, and asymptote. Show the x- and y-intercepts on the graph.
 (a) $f(x) = 2^{-x} + 4$ (b) $g(x) = \log_3(x + 3)$

2. Find the domain of the function.
 (a) $f(t) = \ln(2t - 3)$ (b) $g(x) = \log(x^2 - 1)$

3. (a) Write the equation $6^{2x} = 25$ in logarithmic form.
 (b) Write the equation $\ln A = 3$ in exponential form.

4. Find the exact value of the expression.
 (a) $10^{\log 36}$ (b) $\ln e^3$ (c) $\log_3 \sqrt{27}$
 (d) $\log_2 80 - \log_2 10$ (e) $\log_8 4$ (f) $\log_6 4 + \log_6 9$

5. Use the Laws of Logarithms to expand the expression.
 (a) $\log\left(\dfrac{xy^3}{z^2}\right)$ (b) $\ln \sqrt{\dfrac{x}{y}}$ (c) $\log \sqrt[3]{\dfrac{x + 2}{x^4(x^2 + 4)}}$

6. Use the Laws of Logarithms to combine the expression into a single logarithm.
 (a) $\log a + 2 \log b$ (b) $\ln(x^2 - 25) - \ln(x + 5)$ (c) $\log_2 3 - 3 \log_2 x + \frac{1}{2}\log_2(x + 1)$

7. Find the solution of the exponential equation, rounded to two decimal places.
 (a) $3^{4x} = 3^{100}$ (b) $e^{3x-2} = e^{x^2}$ (c) $5^{x/10} + 1 = 7$ (d) $10^{x+3} = 6^{2x}$

8. Solve the logarithmic equation for x.
 (a) $\log(2x) = 3$ (b) $\log(x + 1) + \log 2 = \log(5x)$
 (c) $5 \ln(3 - x) = 4$ (d) $\log_2(x + 2) + \log_2(x - 1) = 2$

9. Use the Change of Base Formula to evaluate $\log_{12} 27$.

10. The initial size of a culture of bacteria is 1000. After 1 hour the bacteria count is 8000.
 (a) Find a function $n(t) = n_0 e^{rt}$ that models the population after t hours.
 (b) Find the population after 1.5 hours.
 (c) After how many hours will the number of bacteria reach 15,000?
 (d) Sketch the graph of the population function.

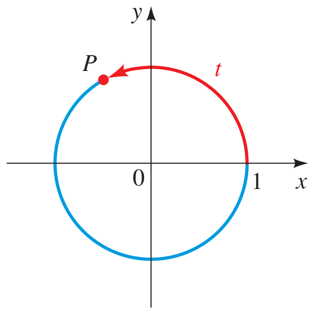
11. Suppose that \$12,000 is invested in a savings account paying 5.6% interest per year.
 (a) Write the formula for the amount in the account after t years if interest is compounded monthly.
 (b) Find the amount in the account after 3 years if interest is compounded daily.
 (c) How long will it take for the amount in the account to grow to \$20,000 if interest is compounded continuously?

12. The half-life of krypton-91 (^{91}Kr) is 10 s. At time $t = 0$ a heavy canister contains 3 g of this radioactive gas.
 (a) Find a function $m(t) = m_0 2^{-t/h}$ that models the amount of ^{91}Kr remaining in the canister after t seconds.
 (b) Find a function $m(t) = m_0 e^{-rt}$ that models the amount of ^{91}Kr remaining in the canister after t seconds.
 (c) How much ^{91}Kr remains after 1 min?
 (d) After how long will the amount of ^{91}Kr remaining be reduced to 1 μg (1 microgram, or 10^{-6} g)?

13. An earthquake measuring 6.4 on the Richter scale struck Japan in July 2007, causing extensive damage. Earlier that year, a minor earthquake measuring 3.1 on the Richter scale was felt in parts of Pennsylvania. How many times more intense was the Japanese earthquake than the Pennsylvania earthquake?

A CUMULATIVE REVIEW TEST FOR CHAPTERS 2, 3, AND 4 CAN BE FOUND AT THE BOOK COMPANION WEBSITE: **www.stewartmath.com**.

CHAPTER 5 TEST



1. The point $P(x, y)$ is on the unit circle in Quadrant IV. If $x = \sqrt{11}/6$, find y.

2. The point P in the figure at the left has y-coordinate $\frac{4}{5}$. Find:

   (a) $\sin t$ **(b)** $\cos t$

   (c) $\tan t$ **(d)** $\sec t$

3. Find the exact value.

   (a) $\sin \dfrac{7\pi}{6}$ **(b)** $\cos \dfrac{13\pi}{4}$

   (c) $\tan\left(-\dfrac{5\pi}{3}\right)$ **(d)** $\csc \dfrac{3\pi}{2}$

4. Express $\tan t$ in terms of $\sin t$, if the terminal point determined by t is in Quadrant II.

5. If $\cos t = -\frac{8}{17}$ and if the terminal point determined by t is in Quadrant III, find $\tan t \cot t + \csc t$.

6–7 ■ A trigonometric function is given.

(a) Find the amplitude, period, phase, and horizontal shift of the function.

(b) Sketch the graph of one complete period.

6. $y = -5 \cos 4x$ 7. $y = 2 \sin\left(\dfrac{1}{2}x - \dfrac{\pi}{6}\right)$

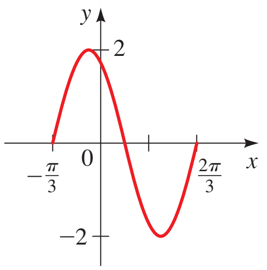
8–9 ■ Find the period, and graph the function.

8. $y = -\csc 2x$ 9. $y = \tan\left(2x - \dfrac{\pi}{2}\right)$

10. Find the exact value of each expression, if it is defined.

    (a) $\tan^{-1} 1$ **(b)** $\cos^{-1}\left(-\dfrac{\sqrt{3}}{2}\right)$

    (c) $\tan^{-1}(\tan 3\pi)$ **(d)** $\cos(\tan^{-1}(-\sqrt{3}))$



11. The graph shown at left is one period of a function of the form $y = a \sin k(x - b)$. Determine the function.

12. The sine curves $y_1 = 30 \sin\left(6t - \dfrac{\pi}{2}\right)$ and $y_2 = 30 \sin\left(6t - \dfrac{\pi}{3}\right)$ have the same period.

    (a) Find the phase of each curve.

    (b) Find the phase difference between y_1 and y_2.

    (c) Determine whether the curves are in phase or out of phase.

    (d) Sketch both curves on the same axes.

13. Let $f(x) = \dfrac{\cos x}{1 + x^2}$.

    (a) Use a graphing device to graph f in an appropriate viewing rectangle.

    (b) Determine from the graph if f is even, odd, or neither.

    (c) Find the minimum and maximum values of f.

14. A mass suspended from a spring oscillates in simple harmonic motion. The mass completes 2 cycles every second, and the distance between the highest point and the lowest point of the oscillation is 10 cm. Find an equation of the form $y = a \sin \omega t$ that gives the distance of the mass from its rest position as a function of time.

15. An object is moving up and down in damped harmonic motion. Its displacement at time $t = 0$ is 16 in.; this is its maximum displacement. The damping constant is $c = 0.1$, and the frequency is 12 Hz.

    (a) Find a function that models this motion.

    (b) Graph the function.

1–8 ■ Verify each identity.

1. $\tan\theta\sin\theta + \cos\theta = \sec\theta$

2. $\dfrac{\tan x}{1 - \cos x} = \csc x\,(1 + \sec x)$

3. $\dfrac{2\tan x}{1 + \tan^2 x} = \sin 2x$

4. $\sin x \tan\left(\dfrac{x}{2}\right) = 1 - \cos x$

5. $2\sin^2(3x) = 1 - \cos(6x)$

6. $\cos 4x = 1 - 8\sin^2 x + 8\sin^4 x$

7. $\left(\sin\left(\dfrac{x}{2}\right) + \cos\left(\dfrac{x}{2}\right)\right)^2 = 1 + \sin x$

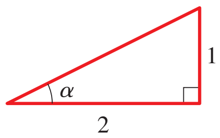
8. Let $x = 2\sin\theta$, $-\pi/2 < \theta < \pi/2$. Simplify the expression

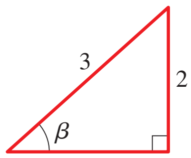
$$\frac{x}{\sqrt{4 - x^2}}$$

9. Find the exact value of each expression.

(a) $\sin 8° \cos 22° + \cos 8° \sin 22°$ **(b)** $\sin 75°$ **(c)** $\sin\dfrac{\pi}{12}$

10. For the angles α and β in the figures, find $\cos(\alpha + \beta)$.





11. Write $\sin 3x \cos 5x$ as a sum of trigonometric functions.

12. Write $\sin 2x - \sin 5x$ as a product of trigonometric functions.

13. If $\sin\theta = -\frac{4}{5}$ and θ is in Quadrant III, find $\tan(\theta/2)$.

14–20 ■ Solve each trigonometric equation in the interval $[0, 2\pi)$. Give the exact value, if possible; otherwise, round your answer to two decimal places.

14. $3\sin\theta - 1 = 0$

15. $(2\cos\theta - 1)(\sin\theta - 1) = 0$

16. $2\cos^2\theta + 5\cos\theta + 2 = 0$

17. $\sin 2\theta - \cos\theta = 0$

18. $5\cos 2\theta = 2$

19. $2\cos^2 x + \cos 2x = 0$

20. $2\tan\left(\dfrac{x}{2}\right) - \csc x = 0$

21. Find the exact value of $\cos\left(2\tan^{-1}\frac{9}{40}\right)$.

22. Rewrite the expression as an algebraic function of x and y: $\sin(\cos^{-1}x - \tan^{-1}y)$.

A CUMULATIVE REVIEW TEST FOR CHAPTERS 5, 6, AND 7 CAN BE FOUND AT THE BOOK COMPANION WEBSITE: **www.stewartmath.com**.