

H. Algebra II Semester II Review

Part 4 – Polynomials

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4.a Polynomial Function Characteristics

Find the polynomial functions with least degree and least integer coefficients that has the following characteristics

1. Rises on the left, falls on the right and has x-intercepts at (-3,0), (-2,0) and (1,0)
2. Rises on the left and right and has x-intercepts (-2,0), (-1,0), (1,0) and (2,0)
3. Falls on the left and right, touches (0,0) and has a complex root (5+i)
4. Falls on the left, rises on the right, passes through (-1,0) and has no complex roots.

Hint: If you are having difficulty with approaching these problems, review the following topics first.

- Polynomial Factor Theorem
- Polynomial graph end behavior
- Fundamental Theorem of Algebra
- Polynomial root multiplicity

4.b Polynomial Long Division

Divide. Report division results as $N(x) \div D(x) = Q(x) + \frac{R(x)}{D(x)}$, where $Q(x)$ is the quotient and $R(x)$ the remainder.

1. $9x^4 - 27x^3 + 53x^2 - 62x + 24 \div (x - 1)$
2. $9x^4 - 27x^3 + 53x^2 - 62x + 24 \div (x^2 - x + 3)$
3. $4x^4 - 2x^3 - 3x - 9 \div (2x^2 - x - 3)$
4. $6x^3 - 5x^2 + 15x - 5 \div (2x^2 - x + 3)$

4.c Solving Polynomial Equations

Find all complex solutions to the following polynomial equations

1. $3x^3 - 81x + 12 = 6(2 - x)$
2. $x - 5x^2 = x^2 - 7x - 14$

Find all complex roots of the following polynomial functions.

3. $f(x) = 4x^3 - 24x^2 + x - 6$
4. $g(x) = 2x^4 - 3x^3 + 16x^2 - 27x - 18$ where $3i$ is a root of $g(x)$

Hint: Check for the following when solving a polynomial

- Always try to factor before resorting to other techniques. Remember higher polynomials can be factored by grouping occasionally.
- Polynomials can sometimes be quadratic analogs and solved by substituting to form a quadratic
- Polynomials with known zeros can be factored using polynomial division.
- Quadratic factors and quadratic analogs can be approached with quadratic formula or completing the square.