Larson 9.6.14 Let $\sum a_n = \sum_{n=0}^{\infty} \frac{3^n}{n!}$

Ratio test: $L = \lim_{n\to\infty} \left| a_{n+1} \cdot \frac{1}{a_n} \right|$

$$L = \lim_{n\to\infty} \left| \frac{3^{n+1}}{(n+1)!} \cdot \frac{n!}{3^n} \right| = \lim_{n\to\infty} \left| \overbrace{\frac{3^{n+1}}{3^n}}^{3} \cdot \overbrace{\frac{n!}{(n+1)!}}^{\frac{1}{n+1}} \right|$$

$$\frac{3^{n+1}}{3^n} = \frac{3^1 \cdot 3^n}{3^n} = 3 \quad , \quad \frac{n!}{(n+1)!} = \frac{n!}{(n+1)\cdot n!} = \frac{1}{n+1}$$

$L = \lim_{n\to\infty} \left| \frac{3}{n+1} \right| = 0$. converges, $L < 1$

Let $\sum a_n = \sum_{n=1}^{\infty} \frac{1}{n^2}$

Ratio Test: $L = \lim_{n\to\infty} \left| \frac{1}{(n+1)^2} \cdot \frac{n^2}{1} \right| = 1$

Inconclusive.

Larson 9.6.23 Let $\sum a_n = \sum_{n=1}^{\infty} \frac{n!}{n \cdot 3^n}$

Ratio test: $L = \lim_{n\to\infty} \left| \frac{(n+1)!}{(n+1)\, 3^{n+1}} \cdot \frac{n \cdot 3^n}{n!} \right|$

$$L = \lim_{n\to\infty} \left| \underbrace{\frac{(n+1)!}{n!}}_{\frac{n+1}{1}} \cdot \underbrace{\frac{3^n}{3^{n+1}}}_{\frac{1}{3}} \cdot \underbrace{\frac{n}{n+1}}_{1} \right| = \infty, \text{ diverges}$$

Larson 9.6.48

Let $\sum a_n = \sum_{n=1}^{\infty} \left(\frac{\ln n}{n}\right)^n$

Root test: $L = \lim_{n\to\infty} \sqrt[n]{\left|\frac{\ln n}{n}\right|^n}$

$L = \lim_{n\to\infty} \frac{\ln n}{n} \leftarrow \frac{\infty}{\infty} = \lim_{n\to\infty} \frac{1/n}{1} = 0$

Converges