Larson 9.8.47 Find the interval of convergence for

(a) $f(x)$ (b) $f'(x)$ (c) $f''(x)$ (d) $\int f(x)\,dx$

(a) $f(x) = \sum_{n=0}^{\infty} \frac{(-1)^{n+1}(x-1)^{n+1}}{n+1} = \sum_{n=1}^{\infty} \frac{(-1)^n (x-1)^n}{n}$

ratio test: $L = \lim_{n \to \infty} \left| \frac{(x-1)^{n+1}}{n+1} \cdot \frac{n}{(x-1)^n} \right| = \lim_{n\to\infty} \left| \underbrace{\frac{(x-1)^{n+1}}{(x-1)^n}}_{\frac{(x-1)}{1}} \cdot \underbrace{\frac{n}{n+1}}_{1} \right|$

$L = |x-1|$

converges if $|x-1| < 1$, $x-1 < 1$, $-(x-1) < 1$

$x < 2$, $x - 1 > -1$

$x > 0$

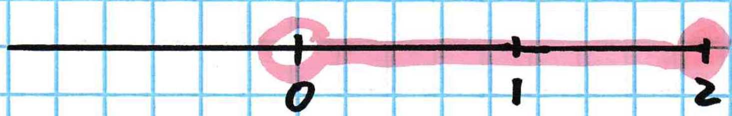
inconclusive if $|x-1| = 1$ or

$x = 0$ or $x = 2$

Left End pt. for $x = 0$, $f(0) = \sum_{n=1}^{\infty} \frac{(-1)^n \cdot (-1)^n}{n} = \sum_{n=1}^{\infty} \frac{1}{n}$, diverges, harmonic

Right End pt. for $x = 2$, $f(2) = \sum_{n=1}^{\infty} \frac{(-1)^n\, 1^n}{n} = \sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ converges, alt. series test

(a) convergence Interval for $f(x)$: $(0, 2]$



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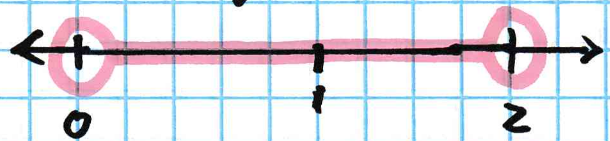
(b) $f'(x) = \left[\sum_{n=1}^{\infty} \frac{(-1)^n (x-1)^n}{n}\right]' = \sum_{n=1}^{\infty} \frac{(-1)^n \cdot n (x-1)^{n-1}}{n}$

$f'(x) = \sum_{n=1}^{\infty} (-1)^n (x-1)^{n-1} = \sum_{n=0}^{\infty} (-1)^{n+1} (x-1)^n$

$= \sum_{n=0}^{\infty} (-1(x-1))^n (-1)$, geometric $r = 1-x$

Converges if $|1-x| < 1$, diverges if $|1-x| \geq 1$

$f'(x)$ convergence interval: $(0, 2)$



(c) $f''(x) = [f'(x)]' = \left[\sum_{n=0}^{\infty} -1 \cdot (\underbrace{\cancel{x-1}}_{1-x})^n\right]' = \sum_{n=0}^{\infty} -1 \cdot n \cdot (\underbrace{\cancel{x-1}}_{1-x})^{n-}$

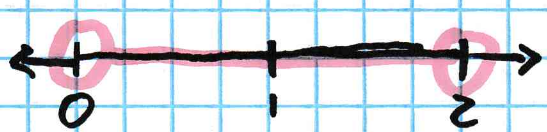
$= \sum_{n=1}^{\infty} -1 \cdot n \cdot (\underbrace{\cancel{x-1}}_{1-x})^{n-1}$

ratio test $L = \lim_{n \to \infty} \left| \frac{(1-x)^n \cdot n+1}{(1-x)^{n-1} \cdot n} \right| = |1-x| = L$

Left End Pt. $f''(0) = \sum_{n=1}^{\infty} n(-1)^n$ diverges, nth term test

Right End Pt. $f''(2) = \sum_{n=1}^{\infty} n$ diverges by nth term test

$f''(x)$ convergence interval: $(0, 2)$



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(d) $\int f(x)\,dx = \int \sum_{n=1}^{\infty} \frac{(-1)^n (x-1)^n}{n}\,dx = \sum_{n=1}^{\infty} \frac{(-1)^n (x-1)^{n+1}}{n(n+1)} = F(x)$

ratio test: $L = \lim_{n\to\infty} \left| \frac{(x-1)^{n+2}}{(n+1)(n+2)} \cdot \frac{n(n+1)}{(x-1)^{n+1}} \right|$

$L = \lim_{n\to\infty} \left| \underbrace{\frac{(x-1)^{n+2}}{(x-1)^{n+1}}}_{\frac{x-1}{1}} \cdot \underbrace{\frac{n(n+1)}{(n+1)(n+2)}}_{1} \right| = \underbrace{|x-1| = L}_{\text{converges for } (0,2)}$

Left End Pt. $F(0) = \sum_{n=1}^{\infty} \frac{-1}{n(n+1)}$ converges, Limit comp. w/ $\sum \frac{1}{n^2}$

Right End Pt. $F(2) = \sum_{n=1}^{\infty} \frac{(-1)^n}{n(n+1)}$ converges, alt. series test

$\int f(x)\,dx$ convergence interval: $[0, 2]$