$f = \underline{1 + x + x^2 + \cdots x^n + \cdots} = \sum_{n=0}^{\infty} x^n = \frac{1}{1-x}$ for $-1<x<1$

Let $f(x) = \frac{1}{1-x}$

find a power series for $\frac{1}{1+x^2}$ ① and $\arctan x$

① $\frac{1}{1+x^2} = \frac{1}{1-(-x^2)} = f(\underline{\underline{-x}}^2) = \sum_{n=0}^{\infty} (-x^2)^n$

$\frac{1}{1+x^2} = \sum_{n=0}^{\infty} (-1)^n x^{2n}$ for $-1<x<1$

②

$\arctan x = \int_0^x \frac{1}{1+t^2}\,dt = \int_0^x \sum_{n=0}^{\infty} (-1)^n t^{2n}\,dt$

$= \int_0^x 1 - t^2 + t^4 - t^6 \cdots dt = t - t^3/3 + t^5/5 \cdots \Big|_0^x$

$\arctan x = x - \frac{x^3}{3} + \frac{x^5}{5} \cdots = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1}$

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generating pi

$\arctan(1) = \frac{\pi}{4}$; $\pi = 4\left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} \cdots\right)$

$\pi = \sum_{n=0}^{\infty} 4(-1)^n \frac{1}{2n+1}$