

H. Algebra II Semester II Review

Part 1 – Linear Functions

Instructor: M. Wolverton

1.a Linear Function Fitting

Find the equation of a line in slope intercept form with the following characteristics.

1. passing through $(\frac{3}{2}, 1)$ and $(5, \frac{5}{2})$
2. Parallel to the line $5x - 3y = 3$ passing through $(-2, 4)$
3. Perpendicular to the line $x + y = 0$ and passing through $(1, 3)$

1.b 2 Variable Linear Systems

Solve the following linear systems of equations by any method. Show your reasoning.

1. $5(1 - x) = 2(x + 3y)$
 $2(x - y) = y - 8$
2. $3(y - 9) - 4x = 11x$
 $7x - y = y - 9$

Try putting the above equations in slope intercept form. Graph the lines from each system separately and check their intersection coordinates. The intersections should match your solution.

1.c 3 Variable Linear Systems

Solve the following three variable system of equations. Elimination method is typically easier for large linear systems.

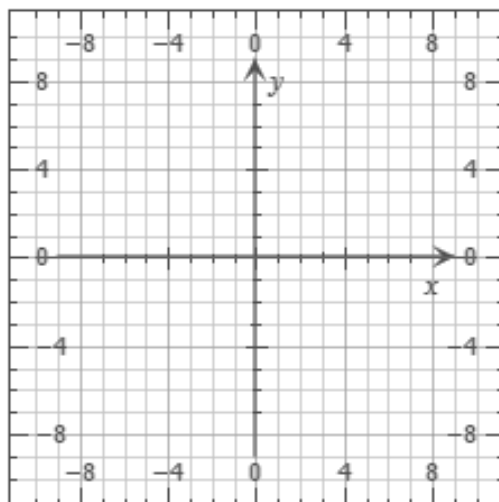
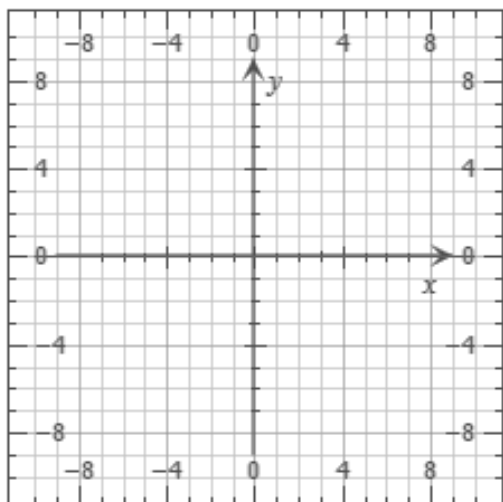
1. $4x - y + 2z = 2$
 $5x - 2y + 2z = 5$
 $-6x + 2y - 5z = 7$

1.d Linear Inequalities and systems

1. The Fahrenheit and Celsius temperature scales are related by $5F = 9C + 160$. If the Fahrenheit temperature is Between -4 degrees and 50 degrees, within what range is the temperature on in Celsius Degrees?

Graph the solutions to the following linear systems of inequalities

2. $x - 4y \geq -8$
 $5x + 2y > -10$
3. $3x + 4y \geq 12$
 $5x - 5y \leq 5$



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Part 2 – Quadratic Functions

Instructor: M. Wolverton

2.a Quadratic Formula

Complete the square and solve for x to derive the quadratic formula.

1. $ax^2 + bx + c = 0$

2.b Solving Quadratic Equations

Solve the following quadratic equations by factoring.

1. $-18x^2 + 33x = 5$

2. $16x^2 - 3 = -22x$

Solve the following quadratic equations by using the quadratic formula or completing the square. Include complex solutions.

3. $x^2 + 6x = -4$

4. $\frac{2x^2+16}{5} = 2x$

5. $\frac{x^2}{2} + 1 = \frac{x}{5}$

6. $5 = 4x(2x + 3)$

2.c Quadratic Function Vertex Properties

Find the Vertex coordinates by completing the square to create vertex form.

1. $f(x) = x^2 + 8x + 6$

Find the Vertex coordinates by first finding the x-intercepts and then averaging them to obtain the vertex x-coordinate.

2. $f(x) = -x^2 - 3x + 10$

Find the Vertex coordinates by using the formula $x = -\frac{b}{2a}$ to obtain the vertex x-coordinate.

3. $f(x) = 2x^2 - 6x + 2$

Characterize each vertex above as a maximum or minimum. If the parabola opens upward ($a > 0$) the vertex is a minimum. If the parabola opens downward ($a < 0$) the vertex is a maximum. Remember the actual maximum / minimum is the y-coordinate of the vertex

2.d Graphing Quadratic Functions

Graph the following quadratic functions. Always find the vertex coordinates first, then find a pair of locations on either side of the vertex. Sketch the parabola once you have coordinates of these three locations.

1. $f(x) = 2(x - 3)^2 + 1$

2. $f(x) = -x^2 + 6x - 10$

2.e Fitting a Quadratic Function

Find the quadratic that passes through the following three points. Remember you can begin the function form $f(x) = ax^2 + bx + c$ then create an equation from each point with a , b and c as unknowns. Solve the three variable system to obtain the values of the coefficients a , b and c so you can construct the quadratic

1. $(-1, 25), (\frac{1}{2}, -7), (3, 3)$

H. Algebra II Semester II Review
Part 3 – Exponent Properties, Complex Numbers, Radical Functions

Instructor: M. Wolverton

3.a Manipulating Expressions with Properties of Exponents

Simplify each expression: combine bases where possible, combine powers where possible, answers should not contain negative or zero exponents and fractions should be reduced.

1. $(3a^3b)^2(ab^3)^3$
2. $(-5k^3) \cdot (-2m+k^2)^2$
3. $\frac{y^{11}}{4z^3} \cdot \frac{8z^7}{y^7}$
4. $\left(\frac{5w^2z^3}{-10w^3z}\right)^2$
5. $\frac{2a^3x^{-2}}{a^2xy^{-1}}$
6. $\left(\frac{2a^{-2}}{b}\right)^{-3}$
7. $\frac{(-3x^2y)^2}{(2a^0xy^2)^3}$

3.b Manipulating Complex Numbers

Simplify the following complex number expressions. Write your answer $a+bi$, where a and b are real numbers.

1. $\left(-3+\frac{1}{2}i\right)^2$
2. $\frac{7+2i}{3-i}$
3. $\frac{3-i}{7+2i}$

3.c Solving Radical Equations

Find all real solutions of the following equations. Watch out for extraneous solutions. Extraneous solutions are most likely to occur when an even power is used during solving. Plug in values to the original equation to check for extraneous solutions.

1. $\sqrt{5-x}-1=x$
2. $\sqrt{4-2x-x^2}=x+2$
3. $\sqrt{5-x}=\sqrt{x}+1$

3.d Properties of Radical Functions

State the domain and range of the following functions.

1. $f(x)=\sqrt{2-3x}-5$
2. $f(x)=-5\sqrt{2x+6}$

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Part 4 - Polynomials

Instructor: M. Wolverton

4.a Polynomial Function Characteristics

Find the polynomial functions with least integer coefficients that has the following characteristics

1. Rises on the left, falls on the right and has x-intercepts at (-3,0), (-2,0) and (1,0)
2. Rises on the left and right and has x-intercepts (-2,0), (-1,0), (1,0) and (0,2)
3. Falls on the left and right, touches (0,0) and has a complex root (5+i)
4. Falls on the left, rises on the right, passes through (-1,0) and has no complex roots.

Hint: If you are having difficulty with approaching these problems, review the following topics first.

- Polynomial Factor Theorem
- Polynomial graph end behavior
- Fundamental Theorem of Algebra
- Polynomial root multiplicity

They are in the McDougal text, but remember: The internet is your friend. Search these topics for material.

4.b Polynomial Long Division

Divide. Report division results as $N(x) \div D(x) = Q(x) + \frac{R(x)}{D(x)}$, where $Q(x)$ is the quotient and $R(x)$ the remainder.

1. $9x^4 - 27x^3 + 53x^2 - 62x + 24 \div (x - 1)$
2. $9x^4 - 27x^3 + 53x^2 - 62x + 24 \div (x^2 - x + 3)$
3. $4x^4 - 2x^3 - 3x - 9 \div (2x^2 - x - 3)$
4. $6x^3 - 5x^2 + 15x - 5 \div (2x^2 - x + 3)$

4.c Solving Polynomial Equations

Find all complex solutions to the following polynomial equations

1. $3x^3 - 81x + 12 = 6(2 - x)$
2. $x - 5x^2 = x^2 - 7x - 14$

Find all complex roots of the following polynomial functions.

3. $f(x) = 4x^3 - 24x^2 + x - 6$
4. $g(x) = 2x^4 - 3x^3 + 16x^2 - 27x - 18$ where $3i$ is a root of $g(x)$

Hint: Check for the following when solving a polynomial

- Always try to factor before resorting to other techniques. Remember higher polynomials can be factored by grouping occasionally.
- Polynomials can sometimes be quadratic analogs and solved by substituting to form a quadratic
- Polynomials with known zeros can be factored using polynomial division.
- Quadratic factors and quadratic analogs can be approached with quadratic formula or completing the square.

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Part 6 – Inverse Functions

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6.a Analytic Definition of Inverse Functions

Determine whether the following pairs of functions are inverse.

1. $f(x) = \frac{x+2}{x-5}$

$$g(x) = \frac{5x+2}{x-1}$$

2. $f(x) = \frac{5}{x-2}$

$$g(x) = \frac{x-2}{5}$$

Find the inverse of each function.

3. $f(x) = 5 + \sqrt{x+1}$

4. $f(x) = \sqrt{\frac{x}{3}} - 6$

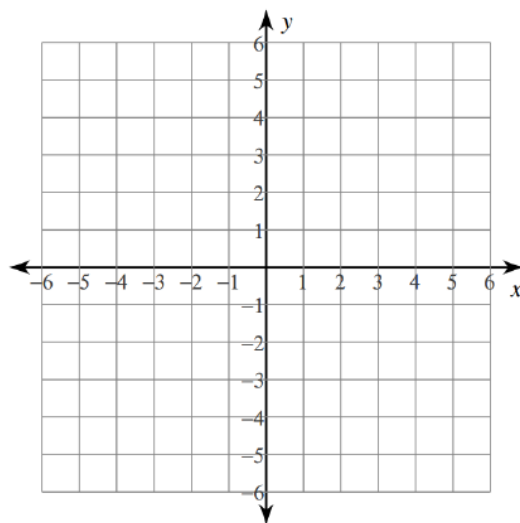
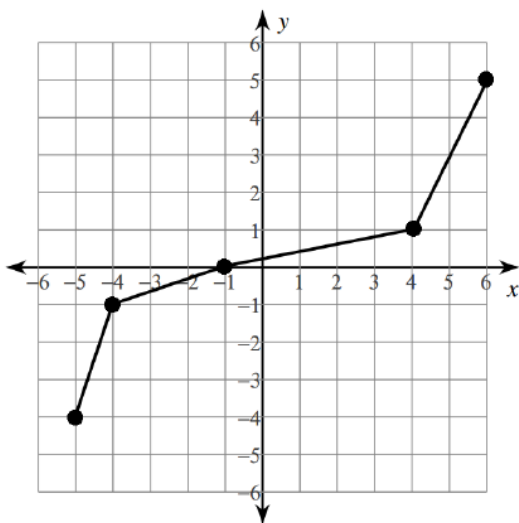
5. $f(x) = \frac{x-3}{x+2}$

6. $f(x) = \sqrt[5]{x^3} = x^{3/5}$

Review the definition of inverse functions (it is a pair of equations), the conditions for existence of an inverse function and the procedure for finding an inverse.

6.b Characteristics of Inverse Functions

Use the graph of the following function to generate a graph of its inverse function on the right.



Recall that if $f(a)=b$ then $f^{-1}(b)=a$. This also means that if the graph of $f(x)$ contains the point (a,b) then the graph of $f^{-1}(x)$ contains the point (b,a) . Because of this, a graph showing both $f(x)$ and $f^{-1}(x)$ should appear to be reflected about the line $y=x$.

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Part 5 - Rational Functions

Instructor: M. Wolverton

5.a Manipulating Rational Expressions and Complex Fractions

Simplify the following rational expressions.

1. $\frac{x}{x-1} - \frac{1}{x+1}$

2. $\frac{2}{x^2+2x} - \frac{1}{x^2+3x+2}$

3. $\frac{\frac{2}{3} - \frac{5}{6}}{\frac{1}{3} + \frac{2}{9}}$

4. $\frac{\frac{1}{x^2} - \frac{1}{y^2}}{\frac{1}{x^2} + \frac{2}{xy} + \frac{1}{y^2}}$

Simplified expressions should only contain one ratio, should not have complex fractions and all fractions should be reduced.

5.b Solving Rational Equations

Solve the following rational equations

1. $\frac{3}{x} + \frac{x}{x-2} = \frac{12}{x^2-2x}$

2. $\frac{2x^2-5x-3}{x^3-2x^2-x+2} = 0$

3. $\frac{3x}{x-2} = \frac{2x+1}{x}$

4. $\frac{x}{x-1} - \frac{1}{x+1}$

Take note of restrictions on x due to division by zero before you begin solving. Screen your answers for these values to avoid extraneous solutions.

5.c Rational Function Characteristics and Graphs

First find the listed characteristics of the following rational functions, then sketch a graph of the function.

1. $f(x) = \frac{3x-1}{3x^2+20x-7}$

2. $f(x) = \frac{4x^2}{x^2+1}$

3. $f(x) = \frac{x^2-5x+4}{x^2-4}$

4. $f(x) = \frac{x^2+3x}{x^2+x-6}$

Characteristics:

- domain
- x-Intercept(s)
- y-intercept
- vertical asymptote(s)
- horizontal asymptote
- hole discontinuities

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Part 7 - Exponential and Logarithmic Functions

Instructor: M. Wolverton

7.a Exponential Function Properties

Find the Domain, Range, and any Asymptotes for each exponential function. Graph each function. Check your graphs with a calculator or graphing utility.

1. $f(x) = 3e^{x+4}$

2. $f(x) = 5 - 2^x$

3. $f(x) = 0.3^x - 2$

4. $f(x) = 3^{-x}$

Typically the domain of an exponential function will be unrestricted (all real numbers), while the range will be restricted. This is because you cannot use an exponent to make an input zero or turn a base negative.

7.b Solving Exponential Equations

Solve the following exponential equations.

1. $8 \cdot 10^{3x} = 12$

2. $e^{3x} = 12$

3. $e^{2x} - 5e^x + 6 = 0$

4. $6(2^{3x-1}) - 7 = 9$

Recall that the inverse of $b^{(x)}$ is $\log_b(x)$, so a logarithm reverses an exponential for solving. You will need to use logarithms in many cases where a variable appears in the exponent during solving.

7.c Logarithmic Function Properties

Find the Domain, Range, and any Asymptotes for each logarithmic function. Graph each function. Check your graphs with a calculator or graphing utility.

1. $f(x) = \log_2(x-3)$

2. $f(x) = 7 - \ln x$

Typically, the domain of a logarithmic function will be restricted while the range will be unrestricted (all real numbers). Recall that $\log_b(n)$ where $n \leq 0$ is undefined. Logarithms with zero or negative arguments are undefined. In other words, you cannot use an exponent to make an input zero or turn a base negative.

7.d Solving Logarithmic Equations

Solve the following logarithmic equations. Exclude extraneous solutions.

1. $3 \ln(5x) = 10$

2. $\log(x-6) = \log(2x+1)$

3. $\ln(x) + \ln(x+3) = 1$

4. $\log_2(x+5) = \log_2(x-1) - \log_2(x+1)$

Recall that the inverse of $\log_b(x)$ is $b^{(x)}$, so exponentiation reverses a logarithm for solving. Extraneous solutions can occur because of the domain restrictions for logarithms (see 8.c).

Review the properties of logarithms (laws of logarithms).

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Part 8 – Trigonometric Functions

Instructor: M. Wolverton

8.a Unit Circle

For the unit circle on the right, each angle θ is a multiple of 30° or 45° .

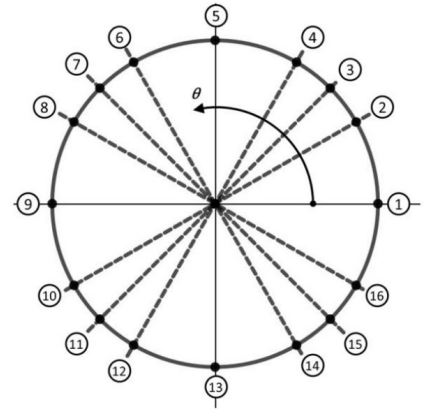
Find the following for each location:

- Angle measure in degrees
- Angle measure in radians
- Cartesian coordinates (x, y)

Define the six trigonometric functions in terms of x and y coordinates on the unit circle.

$$\sin \theta = \quad \cos \theta = \quad \tan \theta =$$

$$\sec \theta = \quad \csc \theta = \quad \cot \theta =$$



8.b Sine and Cosine Function Characteristics

Find the following characteristics of each function, then sketch a graph of the function with one full period. Check your graph with the coordinates of intercepts or extrema.

1. $f(x) = \sin(-3x)$
2. $f(x) = 3 + \sin\left(\frac{1}{2}x\right)$
3. $f(x) = -2 + \cos(4x)$
4. $f(x) = 2 \cos(\pi x) - 3$

Characteristics:

- Period (x spacing between maximums or zeros)
- X interval (between Max. - Center - Min. - Center ...)
- Maximum and Minimum Values (y values)
- Center Line (y value)

Recall that plain $\sin x$ and $\cos x$ have no domain restrictions (domain of all real numbers) but very restricted range of $-1 \leq y \leq 1$ and a period 2π of and an interval between extrema and center of $\frac{\pi}{2}$.

8.c Tangent Function Characteristics

Find the following characteristics of each function, then sketch a graph of the function. Check your graph with the coordinates of intercepts and asymptotes.

1. $f(x) = -\tan\left(\frac{1}{3}x\right)$
2. $\tan(2\pi x)$

Characteristics:

- Period (x spacing between asymptotes or zeros)
- Equations of Asymptotes

Recall that plain $\tan x$ has domain restrictions where $\cos x = 0$ $\left(x = \frac{2n+1}{2}\pi\right)$. These restrictions give the equations of the asymptotes.

Zeros will always occur half way between the asymptotes. $\tan x$ has a period of π .

8.d Solving Trigonometric Equations

Solve the following trigonometric equations to obtain all primary solutions (i.e. $0 \leq x < 2\pi$).

1. $2 \cos(3x) - 1 = 0$
2. $6 \sin^2 x - 7 \sin x + 2 = 0$
3. $\sin^2 x = 3 \cos^2 x$
4. $2 \cos x - 4 \sin x \cos x = 0$

Use the unit circle to locate solutions after you have reduced the complexity of the equation. If using an inverse function, be mindful of the restricted angle outputs.