Trigonometric Integrands pt II

$$\int \sec^m x \tan^n x \, dx$$

$\tan^2 x + 1 = \sec^2 x$

Secant Power is Even → pythag. Trick

tan power is anything

$$\int \sec^4 x \, dx = \int \sec^2 x \underbrace{\sec^2 x \, dx}_{du}$$

$u = \tan x$

$\sec^2 x = u^2 + 1$

$du = \sec^2 x \, dx$

$$= \int u^2 + 1 \, du$$

$$= \frac{1}{3}\tan^3 x + \tan x + C$$

$$\int \sec^4 x \sqrt[3]{\tan x} \, dx$$

$u = \tan x$, $du = \sec^2 x$

$\sec^2 x = u^2 + 1$

$$= \int \underbrace{\sec^2 x}_{u^2+1} \underbrace{\tan^{1/3} x}_{u^{1/3}} \underbrace{\sec^2 x \, dx}_{du} = \int (u^2+1) u^{1/3} \, du$$

$$= \int u^{7/3} + u^{1/3} \, du = \frac{3}{10}\tan^{10/3} x + \frac{3}{4}\tan^{4/3} x + C$$

Tangent power is odd -> Pythag. Trick

Secant pow. is anything

$$\int \tan^3 x \sec x \, dx = \int \underbrace{\tan^2 x}_{u^2-1} \underbrace{\sec x \tan x \, dx}_{du}$$

$$= \int u^2 - 1 \, du \quad \cdots$$

$u = \sec x$ da secondo

$du = \sec x \tan x \, dx$

$\tan^2 x = u^2 - 1$

$$\int \tan^3 x \, dx = \int \underbrace{\tan^2 x}_{u^2-1} \underbrace{\sec^{-1} x}_{u^{-1}} \underbrace{\sec^1 x \tan x \, dx}_{du}$$

$$= \int u - u^{-1} \, du = \frac{1}{2}\sec^2 x - \ln|\sec x| + C$$

$$\int \tan^5 x \sqrt[7]{\sec x} \, dx = \int \underbrace{\tan^4 x}_{} \underbrace{\sec^{-6/7} x}_{u^{-6/7}} \underbrace{\sec x \tan x \, dx}_{du}$$

$u = \sec x$, $du = \sec x \tan x \, dx$

$\tan^4 x = (\tan^2 x)^2 = (u^2-1)^2 = u^4 - 2u^2 + 1$

$$= \int u^{22/7} - 2u^{8/7} + u^{-6/7} \, du$$

$$= \frac{7}{29}\sec^{29/7} x - 2 \cdot \frac{7}{15} \cdot \sec^{15/7} x + 7\sec^{1/7} x + C$$

Secant power is odd → by Parts

$$I = \int \sec^3 x\,dx = \int \sec x \sec^2 x\,dx$$

$u = \sec x \quad du = \sec x \tan x\,dx$

$dv = \sec^2 x\,dx \quad v = \tan x$

$$= \sec x \tan x - \int \underbrace{\tan^2 x}_{\sec^2 x - 1} \sec x\,dx$$

$$= \sec x \tan x - \int \sec^3 x + \int \sec x\,dx$$

$$I = \frac{1}{2}\left(\sec x \tan x + \ln|\sec x + \tan x|\right) + C$$