Trigonometric Integrands

$$\int \sin^m x \cos^n x \, dx$$

I. either sin( ) or cos( ) power is odd
the other power can be anything.

$$\int \sin x \cos^1 x \, dx$$ Let $u = \sin x$ $du = \cos x \, dx$

$$= \int u \, du = \frac{1}{2}\sin^2 x + C$$

$$\int \sin^2 x \cos^1 x \, dx$$ $u = \sin$ $du = \cos x \, dx$

$$= \int u^2 \, du \cdots$$

$$\int \sin^{7/2}(x) \cos^1(x) \, dx \Rightarrow \int u^{7/2} du \cdots$$

$u = \sin x$

$$\int \cos^3 x \, dx = \int \cos^2 x \cos x \, dx = \int (1 - \sin^2 x) \cos x \, dx$$

$$= \int 1 - u^2 \, du$$

$\vdots$

$$\sin^2 x + \cos^2 x = 1$$

$$\boxed{\cos^2 x = 1 - \sin^2 x}$$

$$\int \cos^5 x \, dx = \int \cos^4 x \cos x \, dx = \int (\cos^2 x)^2 \cos x \, dx$$

$$= \int (1 - \underset{u = \sin x}{\sin^2 x})^2 \underbrace{\cos x \, dx}_{du} = \int (1 - u^2)^2 du \cdots$$

odd

$$\int \sin^7 x \sqrt[8]{\cos^7 x}\,dx = \int \sin^6 x \cdot \cos^{7/8} x \,\sin x\,dx$$

$$= \int (\sin^2 x)^3 \cdot \cos^{7/8} x \,\sin x\,dx$$

$1 - \cos^2 x$

Let $u = \cos x$

$-du = \sin x\,dx$

$$= -\int (1-u^2)^3\, u^{7/8}\,du$$

$(1-u^2)^3 =$

$1 - 3u^2 + 3u^4 - u^6$

$u^{7/8}(1-u^2)^3 =$

$u^{7/8} - 3u^{23/8} + 3u^{39/8} - u^{55/8}$

$$= -\int u^{7/8} - 3u^{23/8} + 3u^{39/8} - u^{55/8}\,du$$

$$= -\left[\frac{8}{15}u^{15/8} - 3\,\frac{8}{31}u^{31/8} + 3\,\frac{8}{47}u^{47/8} - \frac{8}{63}u^{63/8}\right]$$

$$= -\,{}^{8}\!/_{15}\cos^{15/8} x + 3\cdot\frac{8}{31}\cos^{31/8} x - 3\,\frac{8}{47}\cos^{47/8} x$$

$$+\,{}^{8}\!/_{63}\cos^{63/8} x + C$$

II. sin() and cos() both are even powers

$\int \cos^2 x\,dx$

$= \frac{1}{2}\int 1+\cos \underbrace{2x}_{u=2x}\,dx$

$\vdots$

$\cos 2x = \cos^2 x - \sin^2 x$

$2\cos^2 x - 1$

$$\boxed{\cos^2 x = \tfrac{1}{2}(1+\cos 2x)}$$

cosine power reducing identity

$\int \sin^2 x\,dx$

$= \frac{1}{2}\int 1-\cos \underbrace{2x}_{u=2x}\,dx$

$\vdots$

$\cos 2x = \cos^2 x - \sin^2 x$

$1 - 2\sin^2 x$

$\sin^2 x = -\tfrac{1}{2}(-1+\cos 2x)$

$$\boxed{\sin^2 x = \tfrac{1}{2}(1-\cos 2x)}$$

sine power reducing identity

8.3.15 $\int \sin^2\alpha \cos^2\alpha \, d\alpha$

$$= \int \frac{1}{2}(1-\cos 2\alpha) \cdot \frac{1}{2}(1+\cos 2\alpha) \, d\alpha$$

$$= \frac{1}{4}\int 1 - \cos^2 2\alpha \, d\alpha$$

$$= \frac{1}{4}\int 1 - \frac{1}{2}(1+\cos 4\alpha) \, d\alpha$$

$$= \int \frac{1}{4} - \frac{1}{8} - \frac{1}{8}\cos 4\alpha \, d\alpha$$

$$= \int \frac{1}{8} - \frac{1}{8}\cos 4\alpha \, d\alpha = \frac{1}{8}\int 1 - \cos \underbrace{4\alpha}_{u} \, d\alpha$$

$\frac{1}{4}du = d\alpha$

$$= \frac{1}{32}\int 1 - \cos u \, du$$

$$= \frac{1}{32}[u - \sin u] = \frac{\alpha}{8} - \frac{1}{32}\sin 4\alpha + C$$