Important Power Series

$f(x)$	First 4 terms	Sigma	Convergence
$\frac{1}{1-x}$	$1+x+x^2+x^3\cdots$	$\sum_{n=0}^{\infty} x^n$	$-1<x<1$
e^x	$1+x+\frac{x^2}{2!}+\frac{x^3}{3!}\cdots$	$\sum_{n=0}^{\infty} \frac{x^n}{n!}$	$-\infty<x<\infty$
$\sin x$	$x-\frac{x^3}{3!}+\frac{x^5}{5!}-\frac{x^7}{7!}\cdots$	$\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$	$-\infty<x<\infty$
$\cos x$	$1-\frac{x^2}{2!}+\frac{x^4}{4!}-\frac{x^6}{6!}\cdots$	$\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}$	$-\infty<x<\infty$

Is e^x actually $1 + x + \frac{x^2}{2!} \cdots$?

$$\frac{d}{dx}[e^x] = e^x$$

note: $\frac{n}{n!} = \frac{1}{(n-1)!}$

$$\frac{d}{dx}\left[1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots\right]$$

$$= 0 + 1 + 2 \cdot \frac{x^1}{2!} + 3\frac{x^2}{3!} + 4\frac{x^3}{4!} \cdots + n \cdot \frac{x^{n-1}}{n!} + \cdots$$

$$= 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots = e^x$$

Is $\sin x$ actually $x - \frac{x^3}{3!} + \frac{x^5}{5!} \cdots$?

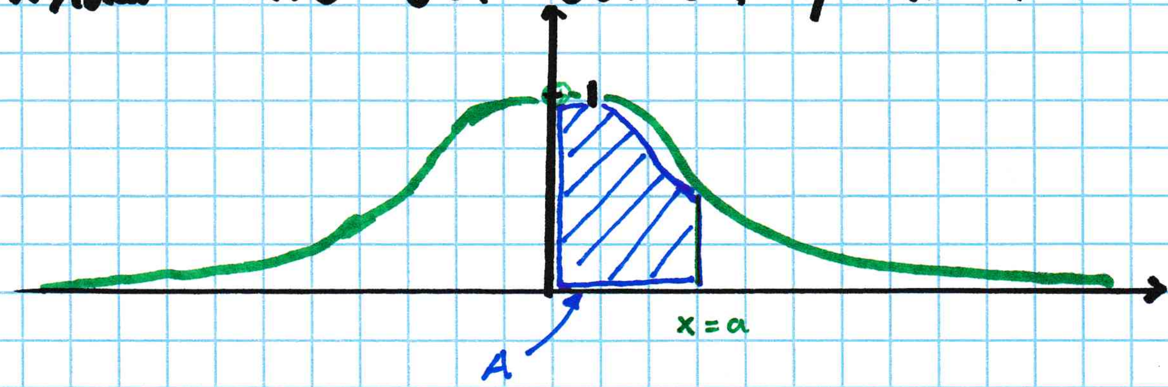
$$\frac{d}{dx}[\sin x] = \cos x$$

$$\frac{d}{dx}\left[x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} \cdots\right] = 1 - \frac{3}{3!} \cdot x^2 + \frac{5}{5!}x^4 - \frac{7}{7!}x^6 \cdots$$

$$= 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} \cdots = \cos x$$

Consider the 'bell curve' parent function e^{-x^2}

Gaussian



$$A = \int_0^a e^{-x^2}\,dx$$

$$e^{(-x^2)} = 1 + (-x^2) + \frac{(-x^2)^2}{2!} + \frac{(-x^2)^3}{3!} \cdots$$

$$= 1 - x^2 + \frac{x^4}{2!} - \frac{x^6}{3!} + \frac{x^8}{4!} \cdots$$

$$= \int_0^a 1 - x^2 + \frac{x^4}{2!} - \frac{x^6}{3!} \cdots \,dx$$

non-elementary Anti derivative

$$= \left[x - \frac{x^3}{3} + \frac{x^5}{5 \cdot 2!} - \frac{x^7}{7 \cdot 3!} + \cdots \right]_0^a$$

$$A = a - \frac{a^3}{3} + \frac{a^5}{5 \cdot 2!} - \frac{a^7}{7 \cdot 3!} + \cdots = \sum_{n=0}^{\infty} (-1)^n \frac{a^{2n+1}}{(2n+1)\,n!}$$

Gaussian Integral : $\int_{-\infty}^{\infty} e^{-x^2} dx$

$$\int_{-\infty}^{\infty} e^{-x^2} dx = 2\int_0^{\infty} e^{-x^2} dx = \cancel{2\int_0^{\infty} e^{-x^2} dx}$$

$$= 2\int_0^{\infty} 1 - x^2 + \frac{x^4}{2!} - \frac{x^6}{3!} \cdots dx$$

$$= 2 \cdot \lim_{b \to \infty} b - \frac{b^3}{3} + \frac{b^5}{5 \cdot 2!} - \frac{b^7}{7 \cdot 3!} + \frac{b^9}{9 \cdot 4!} \cdots =$$

!?

How