Let $f(x)$ be a function that has all orders of derivatives. The Taylor Series for $f(x)$ is defined as

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(c)}{n!}(x-c)^n$$

It is a power series centered at c

If $c=0$ this power series is referred to as a Maclaurin Series.

The Partial Sums of a Taylor Series are called Taylor Polynomials.

Find the Maclaurin $[c=0]$ series for $f(x) = e^x$.

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} \cdot x^n$$

$f(x) = e^x, \quad f(0) = e^0 = 1$

$f'(x) = e^x, \quad f'(0) = e^0 = 1$

$f''(x) = e^x, \quad f''(0) = e^0 = 1$

$f^{(3)}(x) = e^x \quad f^{(3)}(0) = e^0 = 1$

$\vdots$

$f^{(n)}(x) = e^x \quad \underline{f^{(n)}(0) = 1}$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

Conv. Test: $(-\infty, \infty)$ by ratio test

Find the Maclaurin Series for $f(x) = \sin x$.

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} \cdot x^n$$

$f(x) = \sin x$, $f(0) = \sin 0 = 0$

0th deg. poly. $f(x) \approx P_0(x) = 0$

$f'(x) = \cos x$ $f'(0) = \cos 0 = 1$

1st deg Poly P_1 $f \approx P_1(x) = 0 + 1 \cdot x^1$

$f''(x) = -\sin x$ $f''(0) = -\sin 0$

2nd deg Poly P_2 $f \approx P_2(x) = 0 + 1 \cdot x^1 - \frac{0}{2!} x^2$

$f^{(3)}(x) = -\cos x$ $f''(0) = -\cos 0 = -1$

3rd deg Poly P_3 $f \approx P_3 = 0 + \frac{1}{1!} \cdot x^1 - \frac{0}{2!} x^2 - \frac{1}{3!} x^3$

$f^4(x) = \sin x$ ← Starts repeating!

n	$f^{(n)}(0)$
0	0
1	1
2	0
3	-1
4	0
5	1
6	0
7	-1
⋮	⋮

all even n has $f^{(n)}(0) = 0$
odd n alternates sign

$$f(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} \cdots = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$$

$$\sin x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$$

Note: these are the odd terms of e^x power series, but alternating