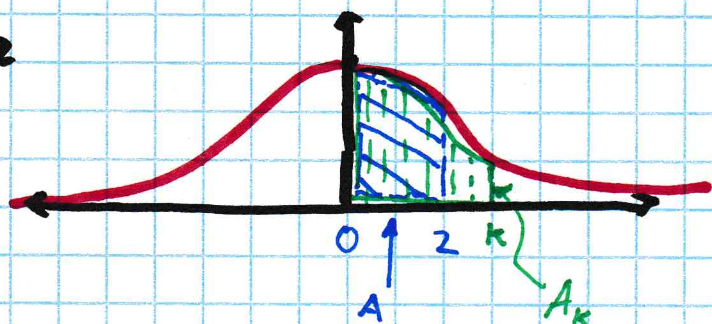
Series intro

$$\frac{1}{3} = .33\bar{3} = \frac{3}{10} + \frac{3}{100} + \frac{3}{1000} + \frac{3}{10^4} \cdots$$

$$e = 2.71828\ldots = 1 + 1 + \frac{1}{1\cdot2} + \frac{1}{1\cdot2\cdot3} + \frac{1}{1\cdot2\cdot3\cdot4} \cdots$$

$$e^x = \frac{x^0}{0!} + \frac{x^1}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} \cdots$$

$f(x) = e^{-x^2}$



$$A = \int_0^2 e^{-x^2} dx \qquad A_K = \int_0^K e^{-x^2} dx = F(K) - F(0)$$

$F(x)$? $\quad \int e^{-x^2} dx$] non Elementary Function

$$e^{(-x^2)} = \frac{(-x^2)^0}{0!} + \frac{(-x^2)^1}{1!} + \frac{(-x^2)^2}{2!} + \frac{(-x^2)^3}{3!} \cdots$$

$$e^{(-x^2)} = 1 + x^2 + \frac{x^4}{2!} - \frac{x^6}{3!} + \frac{x^8}{4!} \cdots$$

$$\int e^{-x^2} dx = \underbrace{x - \frac{x^3}{3} + \frac{x^5}{5\cdot2!} - \frac{x^7}{7\cdot3!}}_{F(x)} \cdots + C$$

Sequences

Sequence - Infinite set of real numbers that can be mapped to the set of non-negative or positive Integers.

Notation

$$\underbrace{a_n}_{\text{sequence 'a'}} = \{ \underbrace{a_0, \overbrace{a_1}^{n=1 \text{ term}}, a_2 \cdots}_{\text{terms}} \}$$

$$a_n = \underbrace{f(n)}_{\text{generating function}} = \{ a_1, a_2, a_3 \cdots \}$$

Defining a sequence

$$b_n = \{ \tfrac{1}{2}, \tfrac{1}{4}, \tfrac{1}{8}, \tfrac{1}{16} \cdots \}$$ Term-by Term Def.

$$b_n = \frac{1}{2^n} = \{ \tfrac{1}{2}, \tfrac{1}{4} \cdots \}$$ Gen. Fn. Def.

$$b_n = \tfrac{1}{2} \cdot b_{n-1}, \quad b_1 = \tfrac{1}{2}$$ Recursive Def.

Gen. Fn. is not Unique

ex. $a_n = \{1, -1, 1, -1 \cdots\}$

$a_n = \underbrace{(-1)^{n+1}}$

not the same function

$a_n = \underbrace{\cos(\pi(n-1))}$

identical sequences

Sequence limits

If $f(x)$ generates a_n and $f(x)$ has a finite Limit as $x \to \infty$ then

$$\lim_{n\to\infty} a_n = \lim a_n = \lim_{x\to\infty} f(x)$$