Series Comparisons

Direct Comparison

- Let $0 \leq a_n \leq b_n$] • all term of both series are pure positive
  - • all terms of a_n are less or equal to corresp. term of b_n

  a_n ↑ Smaller
  b_n ↑ Larger

- IF the Larger series Converges, so does the smaller series.
- IF the Smaller series Diverges, so does the larger series.

Limit Comparison

- Let $a_n > 0$, $b_n > 0$] both series have all terms pure pos. or neg.

if $\lim\limits_{n \to \infty} \dfrac{a_n}{b_n} = L_1$, L is finite, $L \neq 0$

or

$\lim\limits_{n \to \infty} \dfrac{b_n}{a_n} = L_2$ then both series converge

or both series diverge

9.4.11. Let $\sum a_n = \sum_{n=0}^{\infty} \frac{1}{n!}$

• we suspect conv. b/c denom. grows quickly

Let $\sum b_n = \sum_{n=1}^{\infty} \frac{1}{n^2}$, $\sum c_n = \sum_{n=1}^{\infty} \frac{1}{n!} = 1 + \sum a_n$

$\frac{1}{n_2} \geq \frac{1}{n!}$ for $n \geq 4$,

$[\, b_n \geq c_n$ for $n \geq 4 \,]$

n	n^2	$n!$
1	1	1
2	4	2
3	9	6
4	16	24
5	25	120
	⋮	⋮

since $\sum \frac{1}{n^2}$ converges (p-series, $p>1$),

$\sum \frac{1}{n!}$ converges by direct comp.

9.4.9 Let $\sum a_n = \sum_{n=2}^{\infty} \frac{\ln n}{n+1}$,

• we suspect divergence

Let $\sum b_n = \sum_{n=2}^{\infty} \frac{1}{n}$ (diverges, harmonic)

n	$1/n$	a_n
2	$1/2$	$\frac{\ln 2}{3} \approx .23$
3	$1/3$	.27
4	$1/4$	.27
	Smaller	Larger

Since $b_n \leq a_n$ for $n \geq 4$,

and $\sum b$ diverges, then $\sum a$ diverges → by direct comp.

9.4.19

Let $\sum a_n = \sum_{n=1}^{\infty} \frac{2n^2-1}{3n^5+2n+1}$

Let $\sum b_n = \sum_{n=1}^{\infty} \frac{1}{n^3}$ (conv. p-series $p \geq 1$)

$\lim_{n\to\infty} \frac{a_n}{b_n} = 2/3$ $\quad \lim_{n\to\infty} \frac{n^3}{1} \cdot \frac{(2n^2-1)}{(3n^5+2n+1)} = \frac{2}{3}$

Since $\lim_{n\to\infty} \frac{a_n}{b_n} = 2/3 \neq 0$ and $\sum b_n$ converges (p-series $p>1$), $\sum a_n$ converges by Limit comp.

9.4.27

Let $\sum a_n = \sum_{n=1}^{\infty} \sin(1/n)$

Let $\sum b_n = \sum_{n=1}^{\infty} 1/n$ (div., harmonic)

$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{\sin 1/n}{1/n} = \lim_{n\to\infty} \frac{\sin n^{-1}}{n^{-1}} \to \frac{0}{0}$

LHR

$= \lim_{n\to\infty} \frac{\cos n^{-1} \cdot -n^{-2}}{-n^{-2}} = \cos 0 = 1$

$\sum a_n$ diverges by Lim. Comp.