9.1.7

$$a_n = (-1)^{n(n+1)/2} \cdot \frac{1}{n^2}$$

$$\begin{array}{cccccccc} n & & 1 & 2 & 3 & 4 & 5 & \\ a_n = & \{ & -1, & -\frac{1}{4}, & \frac{1}{9}, & \frac{1}{16}, & -\frac{1}{25} & \cdots \} \\ & & a_1 & a_2 & a_3 & & & \end{array}$$

Extra: $\lim a_n = ?$

$$\lim |a_n| = \lim \left|\frac{1}{n^2}\right| = 0 \quad \therefore \quad \lim a_n = 0$$

9.1.13

$$a_{k+1} = \frac{1}{2} a_k, \quad a_1 = 32$$

$$a_n = \{32, 16, 8, 4, 2 \cdots\} = \underline{f(n)}$$

Extra Find $\lim a_n$;

$$f(n) \overset{?}{=} 2^{-n} \qquad f(1) = \frac{1}{2} \qquad a_1 \neq 1/2$$

$$f(n) \overset{?}{=} 64 \cdot 2^{-n} \qquad f(1) = 32 \qquad a_1 = 32$$

$$f(2) = 64 \cdot \frac{1}{4} = a_2 = 16$$

$$\lim a_n = \lim_{n \to \infty} 64 \cdot \frac{1}{2^n} = 0$$

Let $f(n) = n! = a_n$

Def. $(n+1)! = n! \cdot (n+1)$, $0! = 1$

$0! = 1$

$1! = 0! \cdot 1 = 1$

$2! = 1! \cdot 2 = 2$

$3! = 2! \cdot 3 = 2 \cdot 3$

$4! = 3! \cdot 4 = 2 \cdot 3 \cdot 4$

$\vdots$

$n! = 2 \cdot 3 \cdot 4 \cdots (n-1) \cdot n$

n	n!
0	1
1	1
2	2
3	6
4	24
5	120
6	720
7	5040
8	40320
9	362880
⋮	⋮

⚠ $n!$ Does not exist for $n < 0$ or non-integer n

$X!$ has no derivative and is continuous nowhere

$(a+b)! \neq a! + b!$

If $a > b$ then $a!$ contains $b!$

9.1.31 $\quad \dfrac{10!}{8!} = \dfrac{10 \cdot 9 \cdot 8!}{8!} = 90$

9.1.33 $\quad \dfrac{(n+1)!}{n!} = \dfrac{(n+1) \cdot n \cdot (n-1) \cdots}{n \cdot (n-1) \cdots} = n+1$

9.1.35 $\frac{(2n-1)!}{(2n+1)!} = \frac{(2n-1)!}{(2n+1)\cdot(2n)(2n-1)!} = \frac{1}{(2n+1)(2n)}$