

$$\underline{6.3.78} \quad \frac{dy}{dt} = \frac{3}{20}y - \frac{1}{1600}y^2$$

Logistic

$$= \frac{3}{20}y \left(1 - \frac{20}{3} \cdot \frac{1}{1600}y\right)$$

$$\frac{dy}{dt} = ky \left(1 - \frac{y}{L}\right)$$

$$= \frac{3}{20}y \left(1 - \frac{1}{240}y\right)$$

$$\frac{1}{y(1 - \frac{1}{240}y)} \frac{dy}{dt} = \frac{3}{20} \quad ; \quad \frac{240}{y(240-y)} \frac{dy}{dt} = \frac{3}{20}$$

$$\frac{240}{y(240-y)} = \frac{A}{y} + \frac{B}{240-y}$$

$$240 = A(240-y) + By = y(\cancel{240A} - A + B) + (240A)$$

$$240A = 240 \quad ; \quad A = 1$$

$$A = B = 1$$

$$\int \frac{240}{y(240-y)} dx = \int \frac{1}{y} dx + \int \frac{1}{240-y} dx = \int \frac{3}{20} dt$$

$$= \ln|y| - \ln|240-y| = \frac{3}{20}t + \underline{C}$$

$$= \ln \left| \frac{y}{240-y} \right| \quad ; \quad \frac{y}{240-y} = \underline{a} \cdot e^{\frac{3}{20}t}$$

$$\frac{240}{y} - 1 = a e^{-\frac{3}{20}t}, \quad y = \frac{240}{1 + a e^{-3/20t}}$$

General Solution

6.3.78 cont. Find particular soln. through
(0, 15)

$$y(0) = 15 = \frac{240}{1+a}$$

$$15 + 15a = 240$$

$$a = \frac{240 - 15}{15} = \frac{1}{15} (240 - 15)$$

$$= \frac{240}{15} - 1 = \frac{60 \cdot 4}{15} = 24 - 1 = 23$$

$$y = \frac{240}{1 + 23e^{-3/20t}}$$

Particular Solution

DE FRQ 1.b

$$L = \lim_{x \rightarrow 1} \frac{f(x)}{x^3 - 1} \rightarrow \frac{0}{0}$$

$$\boxed{\frac{dy}{dx} = 1 - y}, f(1) = 0$$

LHR

$$L = \lim_{x \rightarrow 1} \frac{f'}{3x^2} = \boxed{\lim_{x \rightarrow 1} \frac{1 - f(x)}{3x^2} = \frac{1}{3}}$$

FRQ 2015.4.d

$$y' = 2x - y$$

$$y' + y = 2x$$

non separable DE

$$\boxed{y = 2x - 2} \quad \checkmark$$

check

$$y' = 2 = 2x - (2x - 2)$$

find a solution of
form $y = mx + b$

$$y' = m = mx + b$$

$$y'' = 0$$

$$y'' = 2 - y'$$

$$y'' = 2 - m = 0$$

$$m = 2$$

$$y' = 2x - y = 2x - mx - b$$

$$y' = -b = 2, \quad b = -2$$