

9.6.19 Let  $\sum a_n = \sum_{n=1}^{\infty} \frac{2^n}{n^2}$  ratio test:

$$L = \lim_{n \rightarrow \infty} \left| a_{n+1} \cdot \frac{1}{a_n} \right| = \lim_{n \rightarrow \infty} \frac{2^{n+1}}{(n+1)^2} \cdot \frac{n^2}{2^n}$$

$$= \lim_{n \rightarrow \infty} \frac{n^2}{(n+1)^2} \cdot \frac{2^{n+1}}{2^n} = 1 \cdot 2$$

Since  $\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| > 1$   $\sum a_n$  diverges by ratio test.

*(Red annotations: A bracket under  $\frac{2^{n+1}}{2^n}$  with an arrow pointing to  $\frac{2^1 \cdot 2^n}{2^n}$ )*

Ratio test is Inconclusive for  $n^p$  type Gen. Functions such as Rational & Power functions (i.e. anything you would test by Lim. Comp.)

Ex  $\sum_{n=1}^{\infty} \frac{1}{n^3}$   $L = \lim_{n \rightarrow \infty} \frac{1}{(n+1)^3} \cdot \frac{n^3}{1} = 1$ , inconc.

$\sum a_n = \sum_{n=1}^{\infty} \frac{2n^2 - 5n + 4}{5n^4 + 2n^2 - 1}$   $L = \lim_{n \rightarrow \infty} a_{n+1} \cdot \frac{1}{a_n} = \frac{2}{5} \cdot \frac{5}{2} = 1$

$\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^2+1}}$   $L = \lim_{n \rightarrow \infty} \frac{1}{\sqrt{(n+1)^2+1}} \cdot \frac{\sqrt{n^2+1}}{1} = 1$

9.6.23  $\sum_{n=1}^{\infty} \frac{n!}{n \cdot 3^n}$  ratio test

$$L = \lim_{n \rightarrow \infty} \frac{(n+1)!}{n!} \cdot \frac{n}{n+1} \cdot \frac{3^n}{3^{n+1}} = \lim_{n \rightarrow \infty} \frac{n+1}{1} \cdot 1 \cdot \frac{1}{3}$$

$L = \infty$ . Diverges