9.8.19 Find Interval of Conv.

$$f(x) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1} x^n}{4^n} = \sum_{n=1}^{\infty} (-1)\left(\frac{-x}{4}\right)^n$$

f(x) is a geometric series: conv. if $\left|\frac{x}{4}\right| < 1$

$\frac{x}{4} < 1$; $\frac{-x}{4} < 1$ $(-4, 4)$

9.8.27 $f(x) = \sum_{n=1}^{\infty} \frac{n}{n+1} (-2x)^{n-1}$

ratio test: $L = \lim_{n\to\infty} \left| \frac{n+1}{n+2} (-2x)^n \cdot \frac{n+1}{n} \frac{1}{(-2x)^{n-1}} \right|$

$$= \lim_{n\to\infty} \left| \underbrace{\frac{(n+1)(n+1)}{(n+2)\cdot n}}_{1} \cdot \underbrace{\frac{(2x)^n}{(2x)^{n-1}}}_{2x} \right| = |2x|$$

f(x) conv. if $|2x| < 1$; $\left(-\frac{1}{2}, \frac{1}{2}\right)$

LH EndPoint $x = -1/2$: $\sum_{n=1}^{\infty} \frac{n}{n+1}$, diverges $\lim_{n\to\infty} a_n \neq 0$

RH EndPoint $x = 1/2$: $\sum_{n=1}^{\infty} \frac{n}{n+1} (-1)^{n-1}$, ,

Interval of Conv. $(-1/2, 1/2)$

Thm 9.21 restated

Suppose $f(x) = \sum_{n}^{\infty} a_n (x-c)^n$] f(x) has a power series centered @ c

① f(x) is differentiable & integrable & continuous within its interval of conv.

② f'(x) and $\int f\,dx$ have the same center and radius

③ f'(x) and $\int f\,dx$ may have different converg. at end points

9.8.47 $\quad f(x) = \sum_{n=0}^{\infty} \frac{(-1)^{n+1}}{n+1} (x-1)^{n+1}$

Ratio test: $\lim_{n \to \infty} \left| \frac{1}{n+2} \cdot (x-1)^{n+2} \cdot \frac{n+1}{1} \cdot \frac{1}{(x-1)^{n+1}} \right|$

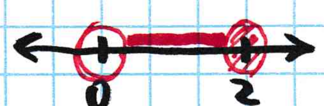
$= \lim_{n \to \infty} \left| \underbrace{\frac{n+1}{n+2}}_{1} \cdot \underbrace{\frac{(x-1)^{n+2}}{(x-1)^{n+1}}}_{x-1} \right| = |x-1|.$

$f(x)$ conv. if $|x-1| < 1$; $(0, 2)$

LH End: $x = 0$; $\sum_{n=0}^{\infty} \frac{1}{n+1}$ diverges, harmonic

RH End: $x = 2$; $\sum_{n=0}^{\infty} \frac{(-1)^{n+1}}{n+1}$ conv. by alt. series test.

$f(x)$ interval: $(0, 2]$



Center: 1
Radius: 1

$f'(x) = \left[-(x-1) + \frac{1}{2}(x-1)^2 - \frac{1}{3}(x-1)^3 + \frac{1}{4}(x-1)^4 \cdots \right]'$

$= -1 + \frac{2}{2}(x-1)^1 - \frac{3}{3}(x-1)^2 - \frac{4}{4}(x-1)^3 + \cdots$

$= \sum_{n=0}^{\infty} (-1)^{n+1} (x-1)^n$

$f'(x)$ is geometric, conv. if $|x-1| < 1$, $(0, 2)$

9.8.47 cont.

$$f''(x) = \left[\sum_{n=0}^{\infty} (-1)^{n+1} (x-1)^n\right]'$$

$$f''(x) = \sum_{n=0}^{\infty} (-1)^{n+1} \cdot n\,(x-1)^{n-1} = \sum_{n=1}^{\infty} (-1)^{n+1} \cdot n(x-1)^{n-1}$$

(!) shift because $n=0$ term is 0

LH End: $x=0$ $\quad \sum_{n=1}^{\infty} n \cdot (-1)^{n-1}$, diverges by $\lim_{n\to\infty} a_n \neq 0$

RH End: $x=2$ $\quad \sum_{n=1}^{\infty} n$, diverges $\lim_{n\to\infty} a_n \neq 0$

f'' interval: $(0, 2)$

$$F = \int f\,dx = \int -(x-1) + \tfrac{1}{2}(x-1)^2 - \tfrac{1}{3}(x-1)^3 + \cdots dx$$

$$F = C + \tfrac{1}{2}(x-1)^2 + \tfrac{1}{2}\cdot\tfrac{1}{3}(x-1)^3 - \tfrac{1}{3}\cdot\tfrac{1}{4}(x-1)^4 + \cdots$$

$$F = C + \sum_{n=0}^{\infty} \frac{(-1)^{n+1}}{(n+1)(n+2)} \cdot (x-1)^{n+2}$$

LH End $x=0$ $\quad \sum_{n=0}^{\infty} \frac{-1}{(n+1)(n+2)}$ Conv. by Lim. Comp. w/ $\sum \frac{1}{n^2}$

RH End $x=2$ $\quad \sum_{n=0}^{\infty} \frac{(-1)^{n+2}}{(n+1)(n+2)}$ Conv. by alt. series test

$F(x)$ interval: $[0, 2]$