A polynomial [function] of degree N is defined as

$$P(x) = \sum_{n=0}^{N} a_n x^n = a_0 + a_1 x + a_2 x^2 \cdots + a_N x^N$$

coefficients

A Power Series is a "Polynomial of infinite Degree"

Centered @ $x=0$

$$f(x) = \sum_{n=0}^{\infty} a_n x^n = a_0 + a_1 x + a_2 x^2 \cdots$$

A polynomial centered at $x = c$ is defined

$$P(x) = a_0 + a_1(x-c) + a_2(x-c)^2 + \cdots + a_N(x-c)^N$$

A Power Series with center $x = c$ is defined

$$f(x) = \sum_{n=0}^{\infty} a_n (x-c)^n$$

Definition of a Power Series

Note: the partial Sums of Power Series are Polynomials

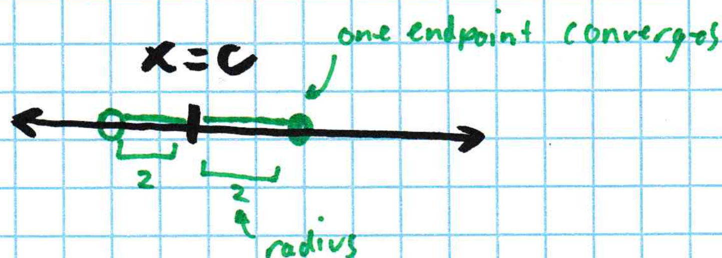
Power Series Convergence depends on values of x.

- Power Series Always converge for $x=c$
- An interval of convergence centered at $x=c$ will have size r, where r is the radius of convergence.

$$0 \le r \quad [r \text{ can be infinite}]$$

- The endpoints of the Interval of Convergence ~~and~~ can independantly converge or diverge.

ex.



Note: Power Series have one unbroken continuous interval of convergence centered at c.

9.8.13 Find the Interval of Conv.

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n} \cdot x^n$$

Note: $c=0$

ratio Test $L = \lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \left|\frac{x^{n+1}}{n+1} \cdot \frac{n}{x^n}\right|$

$$L = \lim_{n\to\infty} \left| \underbrace{\frac{n}{n+1}}_{1} \cdot \underbrace{\frac{x^{n+1}}{x^n}}_{x} \right| = |x|$$

Series Conv. [ratio test] if $|x| < 1$; $\underbrace{(-1, 1)}$

Radius 1
Center 0

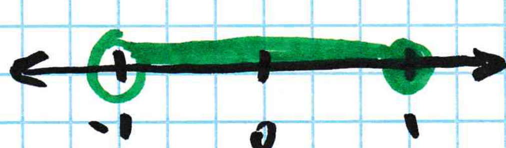
Series Div. [ratio test] if $|x| > 1$

IF $x = 1$ or -1 ratio test is inconclusive(!)

for $x = 1$ $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ conv. by alt. ser. test

for $x = -1$ $\sum_{n=1}^{\infty} \frac{(-1)^n}{n} \cdot (-1)^n = \sum_{n=1}^{\infty} \frac{1}{n}$ Div (harmonic)

Interval of Convergence:



$(-1, 1]$