

PE.mcs

* $y = x^2 - x$

* Speed = $2\sqrt{10}$ const.

$\frac{dx}{dt} > 0$ for all t . find $\frac{dy}{dt}$ @ $(2, 2)$
 x' y'

* $(2\sqrt{10})^2 = 40 = x'^2 + y'^2$

* $\frac{dy}{dt} = 2x \frac{dx}{dt} - \frac{dx}{dt} \Rightarrow y' = x'(2x-1)$

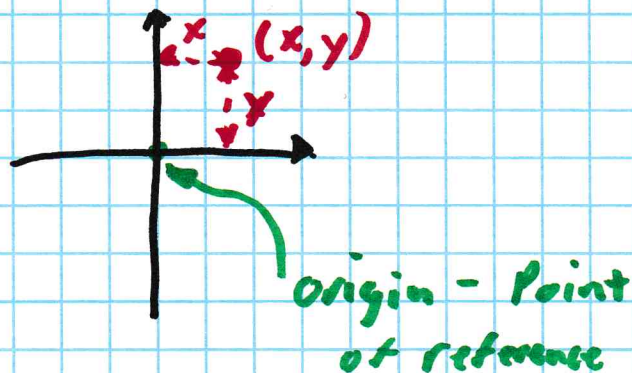
$x' = \frac{1}{2x-1} y'$; $40 = \left(\frac{1}{2x-1}\right)^2 y'^2 + y'^2$
@ $(2, 2)$
 $y'^2 \left(\frac{10}{9}\right) = 40$; ~~$40 = \frac{1}{10} y'^2 + y'^2$~~

$y'^2 = 40 \cdot \frac{9}{10} = 36$; $y' = \pm 6$

Polar Coordinates



Cartesian Coordinates



r - dist to pole

- if r is $\ominus$, π rotation to point

θ - Polar angle

- radians (typically)

- $\oplus$ is counter clockwise

- $\ominus$ is clockwise

- $\mathbb{R}$ Domain, ^{Directions} angles dependent



Every location Has multiple coordinates

Conversions

$$r^2 = x^2 + y^2$$

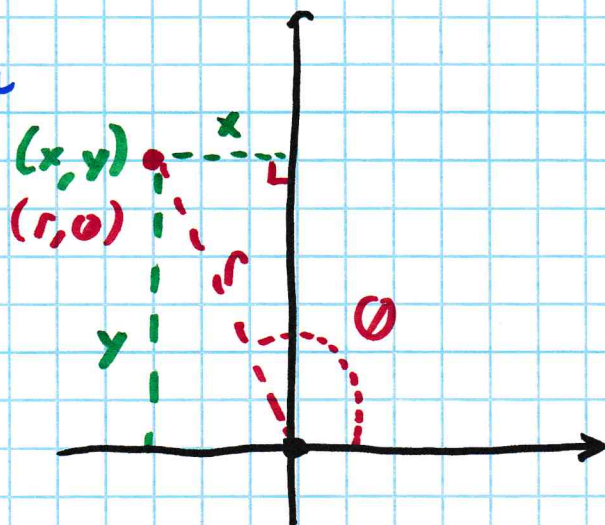
$$r \cdot \cos \theta = x$$

$$r \cdot \sin \theta = y$$

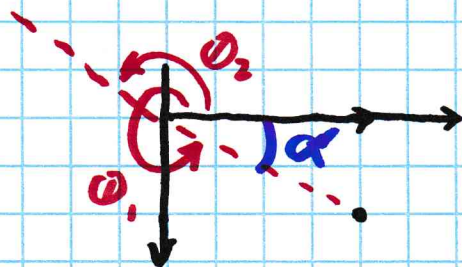
$$\tan \theta = \frac{y}{x}$$

Alignment Convention

- Pole & origin are coincident
- Polar axis is along $x+$



10.4.14 $(x, y) = (4, -2)$ find $z(r, \theta)$, $0 \leq \theta < 2\pi$

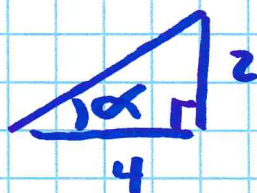


$$r^2 = 16 + 4 = 20$$

$$r = \pm \sqrt{20} = \pm 2\sqrt{5}$$

$$\theta_1 = 2\pi - \alpha$$

$$\theta_2 = \theta_1 - \pi$$



$$\tan \alpha = \frac{1}{2}, \quad \alpha = \arctan \frac{1}{2}$$

$$(2\sqrt{5}, 2\pi - \arctan \frac{1}{2}), \quad (-2\sqrt{5}, \pi - \arctan \frac{1}{2})$$

Polar Curves - typical convention $r = f(\theta)$



$dr/d\theta$: rate of change of radius with respect to angle

$r = c$ is a circle. $dr/d\theta = 0$ means radius and tangent are $\perp$

Parameterization

$$x(\theta) = f(\theta) \cdot \cos \theta, \quad y(\theta) = f(\theta) \cdot \sin \theta$$

• tangent slope: $dy/dx = \frac{dy/d\theta}{dx/d\theta}$

All other parametric calculus holds same meaning