8.5.34 - repeated linear

$$\int \frac{6x^2+1}{x^2(x-1)^3}\,dx$$

$$\frac{6x^2+1}{x^2(x-1)^3} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-1} + \frac{D}{(x-1)^2} + \frac{E}{(x-1)^3}$$

x^3-3x^2+3x-1 $\qquad$ x^2-2x+1

$$6x^2+1 = A(x(x-1)^3) + B(x-1)^3 + Cx^2(x-1)^2 + Dx^2(x-1) + Ex^2$$

$$6x^2+1 = x^4(A+C) + x^3(-3A+B-2C+D) + x^2(3A-3B+C-D+E) + x(-A+3B) + (-B)$$

$$A+C=0$$
$$-3A+B-2C+D=0$$
$$3A-3B+C-D+E=6$$
$$-A+3B=0$$
$$-B=1$$

$\Rightarrow$

A	B	C	D	E	#
1	0	1	0	0	0
-3	1	-2	1	0	0
3	-3	1	-1	1	6
-1	3	0	0	0	0
0	-1	0	0	0	1

$A=-3$
$B=-1$
$C=3$
$D=-2$
$E=7$

$$\int \frac{6x^2+1}{x^2(x-1)^3}\,dx = -3\ln|x| + \frac{1}{x} + 3\ln|x-1| + \frac{2}{x-1} - \frac{7/2}{(x-1)^2} + C$$

8.5.20 $x^3+x^2-x-1 = x^2(x+1)-1(x+1) = (x+1)^2(x-1)$

repeated linear

$$\frac{4x^2}{x^3+x^2-x-1} = \frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{C}{x-1}$$

$$4x^2 = A(x+1)(x-1) + B(x-1) + C(x+1)^2$$

$$4x^2 = x^2(A+C) + x(B+2C) + (-A-B+C)$$

$$\begin{aligned} A + C &= 4 \\ B + 2C &= 0 \\ -A - B + C &= 0 \end{aligned} \Rightarrow \left[\begin{array}{cccc} 1 & 0 & 1 & 4 \\ 0 & 1 & 2 & 0 \\ -1 & -1 & 1 & 0 \end{array}\right] \Rightarrow \begin{aligned} A &= 3 \\ B &= -2 \\ C &= 1 \end{aligned}$$

$$\int \frac{4x^2}{x^3+x^2-x-1}\,dx = 3\ln|x+1| + 2\,\frac{1}{x+1} + 1\ln|x-1| + C$$

8.5.31 irreducible quadratic

$$\int_1^2 \frac{x+1}{x(x^2+1)}\,dx$$

$$\frac{x+1}{x(x^2+1)} = \frac{A}{x} + \frac{Bx+C}{x^2+1}$$

$$x+1 = A(x^2+1) + Bx^2 + Cx$$

$$x+1 = x^2(A+B) + x(C) + (A)$$

$$A+B=0$$

$$C=1$$

$$A=1 \;;\; B=-1$$

$$\int_1^2 \frac{x+1}{x(x^2+1)}\,dx = A\int_1^2 \frac{1}{x}\,dx + B\int_1^2 \frac{x}{x^2+1}\,dx + C\int_1^2 \frac{1}{x^2+1}\,dx$$

$A\int_1^2 \frac{1}{x}\,dx$: $\ln|x|$

$B\int_1^2 \frac{x}{x^2+1}\,dx$: $\frac{B}{2}\ln|x^2+1|$

$C\int_1^2 \frac{1}{x^2+1}\,dx$: $\arctan x$

$$= \ln|x| - \frac{1}{2}\ln|x^2+1| + \arctan x \Big|_1^2$$