

8.5.14. $\int \frac{x^3 - x + 3}{x^2 + x - 2} dx$ } ^① if N degree $\geq$ D degree, divide

~~long division~~

$$\begin{array}{r} x-1 \\ x^2+x-2 \overline{) x^3-x+3} \\ \underline{-(x^3+x^2-2x)} \\ -x^2+x+3 \\ \underline{-(-x^2-x+2)} \\ 2x+1 \end{array}$$

$$\frac{x^3 - x + 3}{x^2 + x - 2} = x - 1 + \frac{2x + 1}{x^2 + x - 2}$$

Note:
v. sub. ...
part. f. not needed,
✓ sub is faster!

$$\frac{2x+1}{(x+2)(x-1)} = \frac{A}{x+2} + \frac{B}{x-1} \quad \text{Partial Frac. Eqn.}$$

$$2x+1 = (x-1)A + (x+2)B \quad \text{Basic Eqn.}$$

$$2x+1 = (A+B)x + (-A+2B) \quad \text{Match coefficients}$$

$$x^1: A+B=2 \quad 3B=3 \Rightarrow B=1, A=1$$

$$x^0: -A+2B=1$$

8.5.12.)

Denom. has only non-repeated linear factors

$$\frac{5x^2 - 12x - 12}{x^3 - 4x} = \frac{5x^2 - 12x - 12}{x(x-2)(x+2)} = \frac{A}{x} + \frac{B}{x-2} + \frac{C}{x+2}$$

$$5x^2 - 12x - 12 = A(x-2)(x+2) + B(x)(x+2) + C(x)(x-2)$$

$$5x^2 - 12x - 12 = (A+B+C)x^2 + (2B-2C)x + (-4A)$$

$$x^2 \quad A+B+C = 5$$

$$x \quad +2B-2C = -12$$

$$x^0 \quad -4A = -12$$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 1 & 5 \\ 0 & 2 & -2 & -12 \\ -4 & 0 & 0 & -12 \end{bmatrix}$$

$$A+B+C = 5$$

|| ref

$$\begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 4 \end{bmatrix} \quad \begin{array}{l} A=3 \\ B=-2 \\ C=4 \end{array}$$

$$\int \frac{5x^2 - 12x - 12}{x^3 - 4x} dx = \int \frac{3}{x} dx + \int \frac{-2}{x-2} dx + \int \frac{4}{x+2} dx$$

$$= 3 \ln|x| - 2 \ln|x-2| + 4 \ln|x+2| + C$$