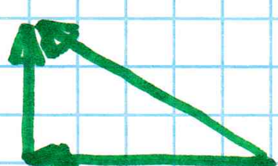
Parametric Derivatives

Let $x(t) = f(t)$, $y(t) = g(t)$

Derivative	2D-motion Interpretation
0th: $(f(a), g(a))$ point	Position at time 'a'
1st 't': $\frac{dx}{dt} = f'(t)$	x-component of velocity; motion is Left if $f' < 0$
$\frac{dy}{dt} = g'(t)$	y-component of velocity; motion is down if $g' < 0$

Speed is defined as $\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$



$\frac{dy}{dx} = \frac{dy}{dt} \Big/ \frac{dx}{dt} = \frac{g'}{f'}(t)$ "trajectory" 'Velocity slope'

tangent slope of graph

2nd deriv.

$d^2x/dt^2 = f''$

x-component of acceleration

⚠ Not concavity

$d^2y/dt^2 = g''$

y-component of acceleration

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left[\frac{dy}{dx}(t)\right] = \overbrace{\frac{dt}{dx}}^{1/f'} \cdot \frac{d}{dt}\left[\frac{dy}{dx}\right]$$

Concavity of graph

$$\boxed{\frac{d^2y}{dx^2} = \frac{d/dt\left[\frac{dy}{dx}\right]}{dx/dt}}$$

⚠ not an acceleration

10.3.13 $\quad x = \cos^3\theta \quad y = \sin^3\theta$

find concavity for $\theta = \pi/4$

$\frac{d^2y}{dx^2}(\pi/4)$

$$\frac{dx}{d\theta} = -3\cos^2\theta\sin\theta$$

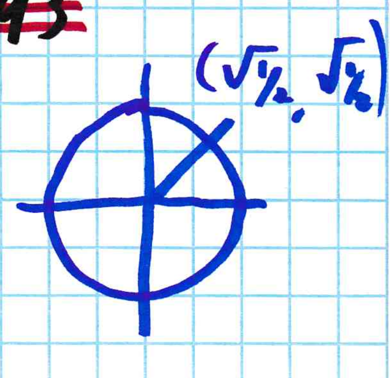
$$\frac{dy}{d\theta} = 3\sin^2\theta\cos\theta$$

$$dy/dx = \frac{3\sin^2\theta\cos\theta}{-3\cos^2\theta\sin\theta} = \cancel{\sin\theta\cos\theta} = -\tan\theta$$

$$\frac{d^2y}{dx^2} = \frac{d}{d\theta}\left[-\tan\theta\right]\cdot\frac{1}{-3\cos^2\theta\sin\theta}$$

$$= \frac{-\sec^2\theta}{-3\cos^2\theta\sin\theta} = \frac{1}{3}\sec^4\theta\csc\theta = \frac{1}{3}\,\frac{1}{\cos^4\theta\sin\theta}$$

$$\frac{d^2y}{dx^2}(\pi/4) = \frac{1}{3}\cdot\frac{1}{1/2^{5/\cancel{4}\,2}} = \frac{2^{5/\cancel{4}\,2}}{3} \approx \cancel{.793}\ 1.886$$



Horizontal & Vertical Tangents

Change in direction for moving object

Horizontal Tangent – $\frac{dy}{dt} = 0$, $\frac{dx}{dt} \neq 0$

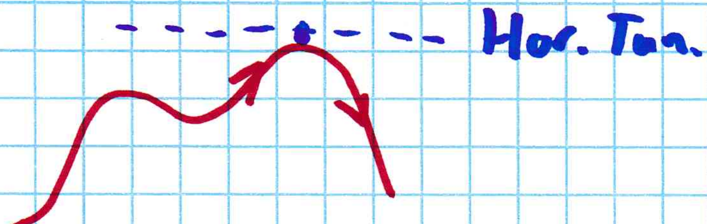
no vertical motion

Horiz. motion is not zero

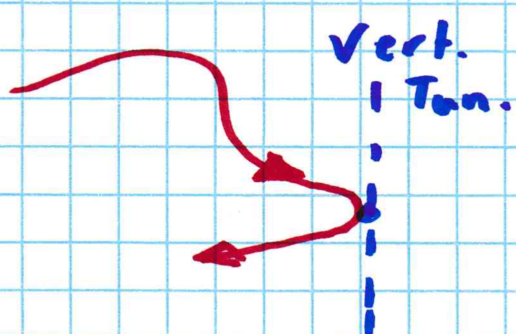
Vertical tangent – $\frac{dy}{dt} \neq 0$, $\frac{dx}{dt} = 0$

still moving up/down

no Horiz. motion

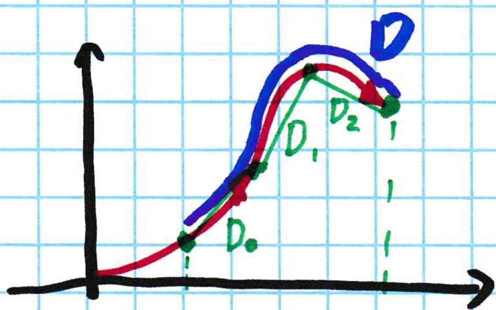


Vertical
Change in direction
Highest / Lowest



Horiz. Change in direction
Farest Left / Right

Arc Length (Parametric)



D - arc Length

D_n - line segments

$$D \approx \sum_n D_n$$

$$D = \lim_{\Delta t \to 0} \sum_n D_n$$

$$D_n = \sqrt{\Delta x_n^2 + \Delta y_n^2}$$

$$\lim_{\Delta t \to 0} \frac{\Delta y}{\Delta x} = dy/dx = \frac{dy}{dt} \Big/ \frac{dx}{dt}$$

$$D_n = \sqrt{\Delta x/\Delta t^2 + \Delta y/\Delta t^2}\ \Delta t$$

Total Distance Travelled

$$\boxed{D = \int_{t_1}^{t_2} \sqrt{\left(dx/dt\right)^2 + \left(dy/dt\right)^2}\ dt}$$

Parametric Arc Length

When $y = f(x)$

$$\boxed{D = \int_{x_1}^{x_2} \sqrt{1 + \left(dy/dx\right)^2}\ dx}$$