$\underline{6.2.7}$ $dy/dx = \sqrt{x}\, y$, $\frac{1}{y} \cdot y' = x^{1/2}$

$\int \frac{1}{y}\, dy = \int x^{1/2} dx$, $\ln|y| = \frac{2}{3} x^{3/2} + C$

$|y| = e^{2/3 x^{3/2} + C} \Rightarrow y = a \cdot e^{2/3 x^{3/2}}$

Logistic Differential Equation

$dy/dt = ky(1 - y/L)$

$y' = ky - \frac{k}{L}y^2$

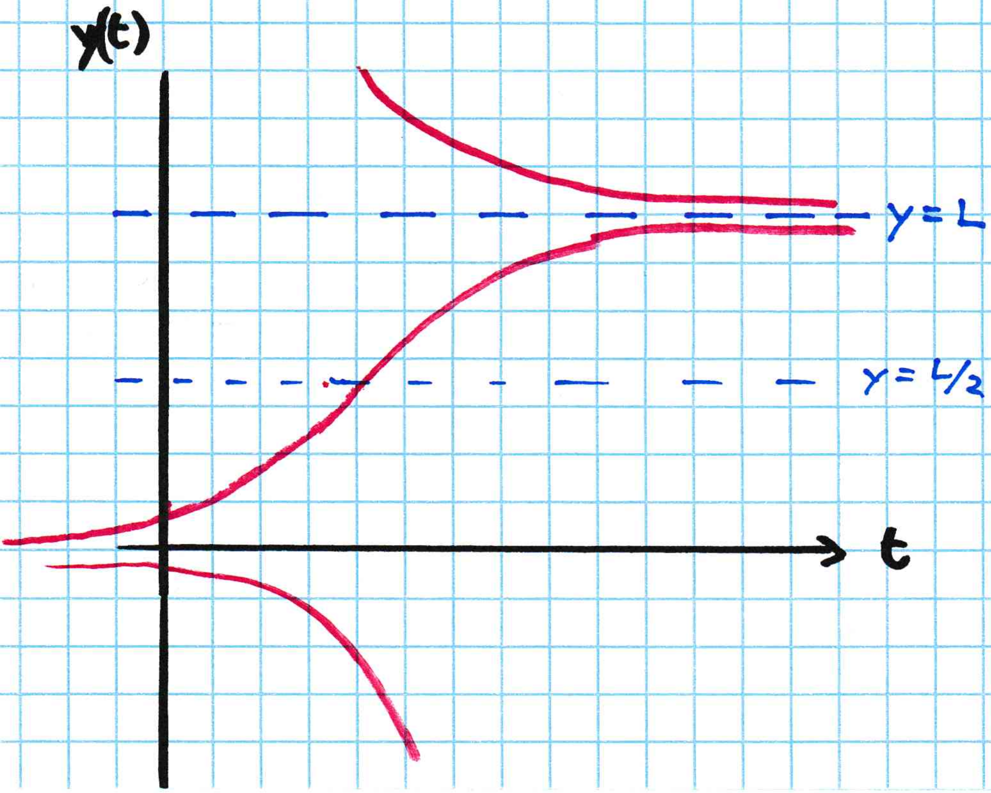
Note: $k, L \geq 0$

L - carrying capacity

k - exp. growth const.

- $y(t)$ has zero rate of change when $y=0$ or $y=L$
- when $0 < y << L$, $y(t)$ resembles exponential growth
- $y'' = ky' - 2\frac{k}{L}yy' = ky'(1 - \frac{2}{L}y)$

  note: $y''=0$ when $y = L/2$, $y(t)$ has max. rate of change, and $y(t)$ has a point of inflection
- When $y > L$, $y(t)$ resembles exponential Decay
- When $y < 0$, $y(t)$ accelerates downward exponentially



Logistic DE solution

$$y' = ky\left(1 - \frac{y}{L}\right), \quad \frac{1}{y\left(1 - y/L\right)} y' = k$$

$$\left(\frac{L}{L}\right) \cdot \frac{1}{y\left(1 - y/L\right)} y' = k, \quad \frac{L}{y(L-y)} y' = k$$

$$\int k\,dt = kt + c = L \int \frac{1}{y(L-y)}\,dy$$

$$\frac{1}{y(L-y)} = \frac{A}{y} + \frac{B}{L-y}$$ Partial Fractions

$$1 = A(L-y) + By$$

$$1 = y(-A+B) + (AL)$$

$$A \cdot L = 1 \; ; \; A = 1/L \; ; \; A = B = 1/L$$

$$L \int \frac{1}{y(L-y)}\,dy = L\left(\frac{1}{L} \int \frac{1}{y}\,dy + \frac{1}{L} \int \frac{1}{L-y}\,dy\right)$$

$$= \ln|y| - \ln|L-y| = kt + c,$$

$$\ln\left|\frac{y}{L-y}\right| = kt + c \Rightarrow \frac{y}{L-y} = ae^{kt}$$

$$\left(\frac{y}{L-y}\right)^{-1} = \left(a \cdot e^{kt}\right)^{-1}, \quad \frac{L-y}{y} = ae^{-kt}$$

$$\frac{L}{y} - 1 = ae^{-kt}, \quad \frac{L}{y} = ae^{-kt} + 1$$

$$\frac{y}{L} = \frac{1}{1+ae^{-kt}}, \quad \boxed{y(t) = \frac{L}{1+ae^{-kt}}}$$