Limit Indeterminate Forms

$$\frac{0}{0}, \quad \pm\frac{\infty}{\infty}, \quad 0\cdot\infty, \quad \infty^0, \quad 1^\infty, \quad 0^0, \quad \infty-\infty$$

L'Hopital's Rule is Directly Applicable

L'Hopital's Rule

if $\lim_{x\to c} \frac{f(x)}{g(x)} \to \frac{0}{0}$ or $\pm\frac{\infty}{\infty}$ then

$$\lim_{x\to c} \frac{f(x)}{g(x)} = \lim_{x\to c} \frac{f'(x)}{g'(x)}$$

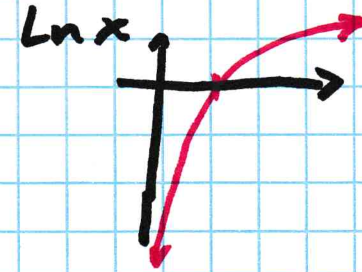
$$\lim_{x\to 0} \frac{\sin x}{x} \to \frac{0}{0}, \quad \lim_{x\to 0} \frac{\sin x}{x} = \lim_{x\to 0} \frac{\cos x}{1} = 1$$

$$\lim_{x\to 0} \frac{\sin x}{x^2} \to \frac{0}{0}, \quad \lim_{x\to 0} \frac{\sin x}{x^2} = \lim_{x\to 0} \frac{\cos x}{2x} = \pm\infty$$

$$\lim_{x\to\infty} \frac{x^{10}}{e^x} \to \frac{\infty}{\infty}, \quad \lim_{x\to\infty} \frac{x^{10}}{e^x} = \lim_{x\to\infty} \frac{10x^9}{e^x} = \lim_{x\to\infty} \frac{10\cdot 9x^8}{e^x}$$

$$\cdots = \lim_{x\to\infty} \frac{!10\ x}{e^x} = \lim_{x\to\infty} \frac{10!}{e^x} = 0$$

$0 \cdot \infty$ type



$$\lim_{x\to 0^+} x \cdot \ln x \to 0(-\infty)$$

$$= \lim_{x\to 0} \frac{\ln x}{x^{-1}} \to \frac{-\infty}{\infty}$$

$$= \lim_{x\to 0} \frac{x^{-1}}{-x^{-2}} = \lim_{x\to 0} -\frac{x^{2}}{x} = -0$$

0^{∞}, 1^{∞}, 0^{0} types

$$L = \lim_{x\to\infty} \left(1+\frac{1}{x}\right)^{x} \to 1^{\infty}$$

$$\ln(L) = \ln\left(\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^{x}\right) = \lim_{x\to\infty} \overset{\infty \cdot 0}{x \cdot \ln\left(1+\frac{1}{x}\right)}$$

$$= \lim_{x\to\infty} \frac{\overset{0}{\ln 1+\frac{1}{x}}}{\underset{0}{x^{-1}}} = \lim_{x\to\infty} \frac{(1+x^{-1})^{-1}\cancel{(-x^{-2})}}{\cancel{(-x^{-2})}}$$

$$= \lim_{x\to\infty} \left(1+\underbrace{x^{-1}}_{0}\right)^{-1} = 1 = \ln L; \quad L = e^{1}$$

$$\lim_{x \to 0^+} x^x \to 0^0, \quad \text{Let } L = \lim_{x \to 0^+} x^x$$

$$\ln L = \lim_{x \to 0^+} x \cdot \ln x = \lim_{x \to 0^+} \frac{\ln x}{x^{-1}} \to \frac{0}{0}$$

$$= \lim_{x \to 0^+} \frac{x^{-1}}{-x^{-2}} = \lim_{x \to 0^+} -x = 0; \quad L = e^0 = 1$$

<u>$\infty - \infty$ type</u>

$$\lim_{x \to 0^+} \csc x - x^{-1} \to \infty - \infty$$

$$= \lim_{x \to 0^+} \frac{1}{\sin x} - \frac{1}{x} = \lim_{x \to 0^+} \frac{x - \sin x}{x \sin x} \to \frac{0}{0}$$

$$= \lim_{x \to 0^+} \frac{1 - \cos x}{\sin x + x \cos x} \Rightarrow \frac{0}{0}$$

$$= \lim_{x \to 0^+} \frac{\sin x}{\cos x + \cos x - x \sin x} \to \frac{0^+}{1 + 1 - 0^+} = 0$$