Introduction to Differential Equations

Differential - Describes how one quantity changes with respect to another

$$dy = f'(x)\,dx \quad \text{if} \quad y = f(x)$$

Differential Equation (not a Differential)

An equation that relates a fuction ($f(x)=y$) and its independant variable(s) (x) to one or more of the function's derivatives (y'', y')

$$x y'' + y y' - xy = y^2 \qquad y' = 7$$

$$\text{Ln}(y'') + (y'')^2 = y$$

$$y' = y$$

Terminology & Classification

- order: highest derivative present
- Solution: a function of function family/set that satisfy (make true) the equation.
- Single vs Multivariate
- Seperable vs nonseperable: Whether DE is solvable by seperation of variables technique

Intro to DE's (2)

- General solution: Function with unconstrained constants that could represent any/all solutions to a D.E.

- Particular Solution: contains only specific constants. One specific function

ex. $y'' = 8$

$y = 4x^2$] particular solution: but Check- $y' = 8x$

$y'' = 8$ ✓

$y = 4x^2 + 2$] ✓

$y = 4x^2 + a$] Solution set, not general

$y = 4x^2 + a_1x + a_0$] General Soln.
- Contains all poss. Particular Solutions

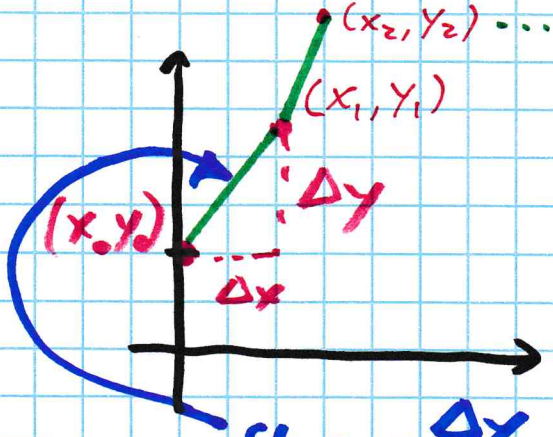
Euler's Method

Numeric method to approximate a particular solution to a 1st order DE. Piecewise Linear Approximation

Ex:

6q.) $y' = x + y$ approximate Part. Soln. that passes through $(0,2)$, 10 steps, step size 0.1.

$\Delta x = .1$



$$\text{slope} = \frac{\Delta y}{\Delta x} = y_1'$$

$$\boxed{\Delta y_n = y_n' \Delta x}$$

$(0,2)$ → $(.1, 2.2)$ → $(.2, 2.43)$

x	0	.1	.2	.3	.4	.5	.6	.7	.8	.9	1
y	2	2.2	2.43	.	.	.					
Δx	.1	→									
Δy	.2	.23	.	.	.						
y'	2	2.3	.	.	.	.					