n 1 2 3 4 5

9.3.5 Let $\sum a_n = \frac{1}{2} + \frac{1}{5} + \frac{1}{10} + \frac{1}{17} + \frac{1}{26} \dots$

3 5 7 9

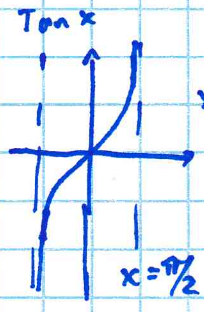
n	n^2
1	1
2	4
3	9
4	16
5	25

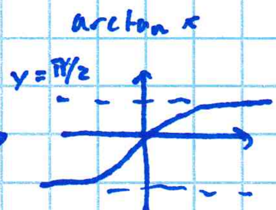
$= \sum_{n=1}^{\infty} \frac{1}{n^2+1}$. Let $f(x) = \frac{1}{x^2+1}$ is pos. dec. and cont. for $x \geq 1$.

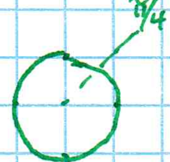
$$\int_1^{\infty} \frac{1}{x^2+1}\,dx = \lim_{b\to\infty} \arctan b - \arctan 1$$

$$= \frac{\pi}{2} - \frac{\pi}{4}. \text{ converges.}$$

$$= \frac{\pi}{4}$$







Since $\int_1^{\infty} f(x)\,dx$ converges $\sum a_n$ converges.

Integral Test Remainder Theorem

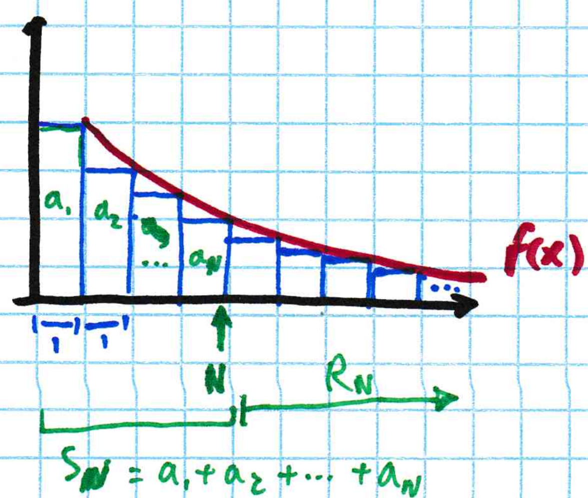
Let $f(n) = a_n$ for $n \geq 1$ and $\sum_{n=1}^{\infty} a_n$ converges by the Integral test.

Let the Nth remainder $R_N = S - S_N$ where $S = \sum_{n=1}^{\infty} a_n$ and $S_N = \sum_{n=1}^{N} a_n$.

$$0 \leq R_N \leq \int_N^{\infty} f(x)\,dx$$

Logic:

$S_N + R_N = S$



9.3.67 Find N for $R_N = S - S_N$ where $R_N \le \frac{1}{1000}$

$\sum a_n = \sum_{n=1}^{\infty} \frac{1}{n^4}$, note $\sum a_n$ conv. by Integral test.

Since $0 \le R_N \le \int_N^{\infty} \frac{1}{x^4}\,dx \le \frac{1}{1000}$

$$\int_N^{\infty} \frac{1}{x^4}\,dx = \underbrace{\lim_{b \to \infty} -\frac{b^{-3}}{3}}_{0} + \frac{N^{-3}}{3} = \frac{1}{3N^3}$$

$\frac{1}{3N^3} \le \frac{1}{1000}$, $3N^3 \ge 1000$, $N \ge \sqrt[3]{\frac{1000}{3}}$

$N \ge 6.933$; $N = 7$ ensures $R_N \le \frac{1}{1000}$

9.3.29 Let $\sum a = \sum_{n=1}^{\infty} \frac{1}{\sqrt[5]{n}} = \sum_{n=1}^{\infty} \frac{1}{n^{1/5}}$.

$\sum a_n$ is a p-series with $p = 1/5$. $\sum a_n$ diverges since $p < 1$.

$$\sum_{n=1}^{\infty} \frac{1}{n} \text{ diverges (harmonic)}$$

$$\sum_{n=1}^{\infty} \frac{1}{n \cdot \sqrt[100]{n}} \text{ converges } \left(p\text{-series } p = 1.01\right)$$

$$\text{Let } \sum a_n = \sum_{n=2}^{\infty} \frac{1}{\underbrace{n \ln(n)}_{f(x) = \frac{1}{x \cdot \ln x}}} \quad \text{Diverges}$$

✓pos. ✓cont. ✓dec.

$$\int_2^{\infty} \frac{1}{x \cdot \ln x}\, dx$$

Let $u = \ln x$

$du = \frac{1}{x}\, dx$

$$\int_{\ln 2}^{\infty} \frac{1}{u}\, du = \ln|u| \Big|_{\ln 2}^{\infty}$$

$$= \underbrace{\lim_{b \to \infty} \ln(\ln b)}_{\infty} - \ln(\ln(2))$$

$$\text{Let } \sum a_n = \sum_{n=2}^{\infty} \frac{1}{n \cdot (\ln n)^2} \quad \text{Converges}$$