9.2.81 $\sum_{n=1}^{\infty} (x-1)^n$ converges if $|x-1| < 1$.

Conv. Interval for x: $(0,2)$

$-(x-1) < 1$ or $x-1 < 1$

$x-1 > -1$

$x > 0$ $x < 2$

Sum for conv. Values of x

Let $\sum_{n=1}^{\infty} (x-1)^n = f(x) = \frac{x-1}{1-x+1} = \frac{x-1}{2-x} = \frac{x-1}{-x+2}$

Geometric Series Convergence Proof

Let $\sum_n b_n = \sum_{n=1}^{\infty} a \cdot r^n$, $S_k = \sum_{n=1}^{k} a r^n$

$\sum b_n$ converges if $\lim_{k \to \infty} S_k$ exists. If $\sum b_n$ converges,

$\sum b_n = \lim_{k \to \infty} S_k = S$.

$$\sum b_n = ar + ar^2 + ar^3 \ldots = S$$

$$S_1 = ar \quad , \quad S_2 = ar + ar^2 \quad , \quad S_3 = ar + ar^2 + ar^3$$

$$r \cdot S_k = (ar + ar^2 + \cdots + ar^{k-1} + ar^k) \cdot r$$

$$r \cdot S_k = \underbrace{ar^2 + ar^3 + \cdots + ar^k + ar^{k+1}}$$

$$S_{k+1} = S_k + ar^{k+1} = ar + ar^2 + \cdots + ar^k + ar^{k+1}$$

$$S_k + ar^{k+1} - ar = \underbrace{ar^2 + \cdots + ar^{k+1}}$$

$$r \cdot S_k = S_k + ar^{k+1} - ar$$

$$S_k(r-1) = ar^{k+1} - ar \quad , \quad S_k = \frac{ar^{k+1}}{r-1} - \frac{ar}{r-1}$$

$\lim_{k \to \infty} S_k$ DNE if $r = 1$ or $|r| > 1$

$\lim_{k \to \infty} S_k = -\frac{ar}{r-1} = \frac{ar}{1-r}$ if $|r| < 1$.

Integral Test for convergence and divergence

if $\sum_{n=1}^{\infty} a_n = \sum_{n=1}^{\infty} f(n)$ then $\int_1^{\infty} f(x)\,dx$ and $\sum a_n$

both converge or both diverge.

Note: $f(n)$ must be

- Positive
- Decreasing for $n \geq 1$
- continuous

ex $\sum_{n=1}^{\infty} \frac{1}{n} = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} \cdots$] Harmonic

$\lim_{n \to \infty} \frac{1}{n} = 0$ (does not diverge by nth term)

note $f(x) = \frac{1}{x}$ is positive, decreasing, and continuous for $x \geq 1$.

$$\int_1^{\infty} \frac{1}{x}\,dx = \lim_{b \to \infty} [\ln b - \ln 1] = \infty, \text{ diverges}$$

$\therefore \sum_{n=1}^{\infty} \frac{1}{n}$ diverges.

Let $\sum_{n=1}^{\infty} \frac{1}{n^2} = \sum a$. $\frac{1}{x^2}$ is pos. dec. & cont. for $x \geq 1$

$$\int_1^{\infty} \frac{1}{x^2} = \lim_{b \to \infty} -b^{-1} + 1 = 1$$

$\therefore \sum a$ converges.

! we do not establish the value of the series sum with the integral test!