Integration By Parts

- Integral inverse of Derivative Product Rule

$$\frac{d}{dx}[f \cdot g] = \frac{df}{dx} \cdot g + f \frac{dg}{dx}$$

$$f \cdot g + C = \int g \frac{df}{dx}\, dx + \int f \frac{dg}{dx}\, dx$$

$$\int \underbrace{f}_{u} \underbrace{\frac{dg}{dx}\, dx}_{dv} = \underbrace{f \cdot g}_{u \cdot v} - \int \underbrace{g}_{v} \underbrace{\frac{df}{dx}\, dx}_{du}$$

$$\int u \, dv = u \cdot v - \int v \, du$$

$$\boxed{\int u \, dv = uv - \int v \, du}$$

Integration By Parts

(IBP) Integration By Parts

$$\int x \cdot \sin x \, dx$$

IBP $u = x$ $du = 1 \cdot dx$

$dv = \sin x \, dx$ $v = \int \sin dx = -\cos x$

$$\int u \, dv = uv - \int v \, du$$

$$\int x \sin x \, dx = -x \cos x - \int -\cos x \, dx$$

$$= -x \cos x + \sin x + C$$

Check $[-x\cos x + \sin x]'$

$$= -\cos x + x \sin x + \cos x = x \sin x$$

$$\int x^2 e^x \, dx$$

IBP $u_1 = x^2$ $du_1 = 2x dx$

$dv_1 = e^x dx$ $v_1 = e^x$

$$= x^2 e^x - \int 2x e^x \, dx$$

IBP $u_2 = 2x$, $du_2 = 2dx$

$dv_2 = e^x$, $v_2 = e^x$

$$= x^2 e^x - 2x e^x + \int 2 e^x \, dx$$

$$= x^2 e^x - 2x e^x + 2 e^x + C$$

$$\int \ln x \, dx$$

IBP $u = \ln x$ $du = \frac{1}{x} dx$

$dv = dx$ $v = x$

$$= x \ln x - \int x \cdot \frac{1}{x} \, dx$$

$$= x \ln x - \int dx = x \ln x - x + C$$

Check $\left[x \ln x - x \right]'$

$$= \ln x + \frac{x}{x} - 1 = \ln x$$

$$\int \ln x \, dx = x \ln x - x + C$$

8.2.16 $\int x^4 \, Lnx \, dx \overset{\text{Let}}{=} I$

op. 1

$u = x^4 \qquad du = 4x^3 dx$

$dv = Lnx \qquad v = xLnx - x$

$$I = x^5 Lnx - x^5 - \underbrace{\int 4x^4 Lnx}_{-4I} - 4x^4 dx \, ;$$

$$5I = x^5 Lnx - x^5 + 4\frac{x^5}{5} + C \implies I = \frac{x^5}{5} Lnx - \frac{x^5}{25} + C$$

op. 2

$u = Lnx \qquad du = \frac{1}{x} dx$

$dv = x^4 dx \qquad v = x^5/5$

$$\cdots = \frac{x^5}{5} Lnx - \int x^4/5 \, dx = \frac{x^5}{5} Lnx - \frac{x^5}{25} + C$$