9.8.82 $f(x)=\sum_{n=0}^{\infty} a_n x^n$ has a radius of conv. ≥ 2
[(0,1) is in domain for certain]

$$\int_0^1 f\,dx = F(1) - F(0) = \underbrace{C + \sum_{n=0}^{\infty} \frac{a_n}{n+1} - C}_{0}$$

True

$$\int f\,dx = \int a_0 + a_1 x + a_2 x^2 \cdots dx = C + a_0 x + a_1 \frac{x^2}{2} + a_2 \frac{x^3}{3} \cdots$$

$$= F(x) = C + \sum_{n=0}^{\infty} a_n \frac{x^{n+1}}{n+1}$$

$$F(1) = C + \sum_{n=0}^{\infty} \frac{a_n}{n+1}, \qquad F(0) = C + \sum_{n=0}^{\infty} 0 = C$$

What functions can be represented by Power Series?

Examine <u>Geometric</u> power series

$$\text{Let } f(x) = \sum_{n=0}^{\infty} x^n = \underline{1 + x + x^2 + x^3 \cdots = \frac{1}{1-x}}$$

$f(x)$ converges for $(-1, 1)$, geometric $|x| < 1$ conv., div. otherwise

for $(-1,1)$, $\underline{f(x) = \frac{1}{1-x}}$

Find Power Series for $g(x) = \frac{1}{x}$

$$\frac{1}{x} = g(x) = \frac{1}{1-(1-x)} = f(1-x) = \sum_{n=0}^{\infty} (1-x)^n = \underbrace{\sum_{n=0}^{\infty} (-1)^n \cdot (x-1)^n}$$

Int. of conv. $(0, 2)$
Cent. $c = 1$, $R = 1$

Find a Power series for $h(x) = \operatorname{Ln} x$

$$\operatorname{Ln} x = \int_1^x \frac{1}{t}\,dt = \int_1^x g(t)\,dt = \int_1^x 1 - (t-1) + (t-1)^2 - (t-1)^3 \cdots dt$$

$$= \left[t - \frac{1}{2}(t-1)^2 + \frac{1}{3}(t-1)^3 \cdots \right]_1^x = \underline{x} - \frac{1}{2}(x-1)^2 + \frac{1}{3}(x-1)^3 \cdots$$

$$\underline{-1} \;\; \cancel{= \sum_{n=0}^{\infty} (-1)^{n+1} \cdot \frac{1}{n+1}(x-1)^{n+1} = \sum_{n=0}^{\infty} \frac{(-1)^{n+1}}{n} \cdot (x-1)^n}$$

$$\cancel{\operatorname{Ln} x = -1 + (x-1) - \frac{1}{2}(x-1)^2 \cdots}$$

$$\operatorname{Ln} x = (x-1) - \frac{1}{2}(x-1)^2 + \frac{1}{3}(x-1)^3 \cdots = \sum_{n=0}^{\infty} \frac{(-1)^n}{n+1}(x-1)^{n+1}$$

$$\operatorname{Ln} 2 = \sum_{n=0}^{\infty} \frac{(-1)^n}{n+1} = \underline{1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} \cdots = \operatorname{Ln} 2}$$