9.9.5 Find Pow. Series $c=5$ for $f=\frac{1}{2-x}$

$$\boxed{\sum_{n=0}^{\infty} ar^n = \frac{a}{1-r}}$$

⚠ $f=\frac{1}{2(1-\frac{x}{2})} = \frac{1}{2}\cdot\frac{1}{1-x/2} = \sum_{n=0}^{\infty}\frac{1}{2}\left(\frac{x}{2}\right)^n$

$c=0$ ⚠

Note: $\sum a_n(x-5)^n$ is centered @ 5

$$f=\frac{1}{2-x}=\frac{1}{2-(x-5+5)}=\frac{1}{2-(x-5)-5}=\frac{1}{-3-(x-5)}$$

$$=\frac{1}{-3\left(1-\left(\frac{x-5}{-3}\right)\right)} = \frac{-1}{3}\cdot\frac{1}{1-\left(\frac{x-5}{-3}\right)} = \sum_{n=0}^{\infty}\frac{-1}{3}\left(\frac{-1}{3}\right)^n(x-5)^n$$

$$=\sum_{n=0}^{\infty}\frac{-1}{3}\left(\frac{x-5}{-3}\right)^n \quad \text{conv. for } \underbrace{\left|\frac{x-5}{-3}\right|<1}_{\text{geom. series test}}\ ; \quad \overset{\text{int. of conv.}}{(2,8)}$$

9.9.13 Find Pow. Series $c=0$ $g=\frac{3x}{x^2+x-2}$

Part. Frac. $\frac{3x}{(x+2)(x-1)}=\frac{A}{x+2}+\frac{B}{x-1}$, $3x=A(x-1)+B(x+2)$

$3x = x(A+B)+(-A+2B)$; $\begin{array}{l}-A+2B=0 \\ A+B=3\end{array}$; $\begin{array}{l}A=2B \\ 3B=3,\ \boxed{B=1, A=2}\end{array}$

$$g=2\frac{1}{x+2}+\frac{1}{x-1} = 2\frac{1}{2\left(1-\left(\frac{-x}{2}\right)\right)}+\frac{1}{-(1-x)}$$

$$g=\frac{1}{1-(-x/2)}-\frac{1}{1-x}=\underbrace{\sum_{n=0}^{\infty}\left(\frac{-x}{2}\right)^n}_{\left(\frac{-1}{2}\right)^n x^n}-\sum_{n=0}^{\infty}x^n \qquad \overset{\text{int.}}{(-1,1)}$$

$$g=\sum_{n=0}^{\infty}\left[\left(\frac{-1}{2}\right)^n-1\right]x^n$$

9.9.23 Find Pow. Series $c=0$ $\quad y = \dfrac{1}{x^2+1}$

$$y = \frac{1}{1-(-x^2)} = \sum_{n=0}^{\infty} (-x^2)^n = \sum_{n=0}^{\infty} (-1)^n x^{2n}$$

conv. $(-1,1)$: geom. ser. $|-x^2| < 1$ cov.

$$\arctan x = \int_0^x \frac{1}{1+t^2}\,dt = \int_0^x \sum_{n=0}^{\infty} (-1)^n t^{2n}\,dt$$

$$= \int_0^x 1 - t^2 + t^4 - t^6 \cdots dt$$

$$= \left[t - \frac{t^3}{3} + \frac{t^5}{5} - \frac{t^7}{7} \cdots \right]_0^x$$

$$\arctan x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} \cdots = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1}$$

(for $-1 < x \leq 1$)

$$\arctan(1) = \overbrace{1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7}}^{\sum (-1)^n \frac{1}{2n+1}} \cdots = \frac{\pi}{4}$$

$$\pi = 4 - \frac{4}{3} + \frac{4}{5} - \frac{4}{7} + \frac{4}{9} \cdots$$

9.9.37

$$\int_0^{1/2} \frac{1}{x} \cdot \arctan x^2 \, dx = a$$

$$\frac{1}{x} \cdot \underline{\underline{\arctan}}(x^2) = \frac{1}{x}\underbrace{\left((x^2) - \frac{(x^2)^3}{3} + \frac{(x^2)^5}{5} + \cdots\right)}_{\arctan(\ )}$$

$$= x - \frac{x^5}{3} + \frac{x^9}{5} - \frac{x^{13}}{7} + \frac{x^{17}}{9} \cdots$$

$$\frac{1}{x} \arctan(x^2) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{4n+1}}{2n+1}$$

$$\int \frac{1}{x} \arctan x^2 \, dx = \int x - \frac{x^5}{3} + \frac{x^9}{5} - \frac{x^{13}}{7} \cdots dx$$

$$\vdots$$

$$\cancel{a = \frac{1}{2} - \frac{(1/2)}{3} +}$$