Def. of Derivative

1. Rate of change: Avg. $$: \frac{f(x_0) - f(x_1)}{x_2 - x_1}$$

$$f(x) = mx + b$$

2. Instantaneous rate

$$m \approx \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$m = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$\frac{d}{dx}[f(x)] = \frac{df}{dx} : = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

Note: $$\lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} \to \frac{0}{0}$$
Differentiability $\Rightarrow$ Existence of tangent slope

Limit

Non-Differentiability

1. What causes limits to not exist?
   - Oscillations $\Rightarrow$ oscillations in the function are non-diff.

2. LH ≠ RH $\Rightarrow$ cusps

3. Infinite limit $\Rightarrow$ vert. tangent line

Discontinuity

All functions are non-diff. if discontinuous at a point.
Elementary Function Classification & Derivatives

<table>
<thead>
<tr>
<th>Elementary functions</th>
<th>Non-Elementary functions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Algebraic</td>
<td>Elem. Transcendental</td>
</tr>
<tr>
<td>Polynomial</td>
<td>Trigonometric</td>
</tr>
<tr>
<td>Root</td>
<td>Inverse Trig.</td>
</tr>
<tr>
<td>Rational</td>
<td>Exponential</td>
</tr>
<tr>
<td>Power</td>
<td>Logarithmic</td>
</tr>
<tr>
<td>Exponential</td>
<td>Hyperbolic Trig.</td>
</tr>
<tr>
<td>Hyperbolic Trig.</td>
<td>Inverse Hyperb.</td>
</tr>
</tbody>
</table>

Algebraic functions can have any finite quantity of combinations of + - * / or _n_ (where n is a rational exponent)

Exponential functions include irrational exponents making the operations non-algebraic.

All compositions (layers) of elementary functions are also elementary functions as long as there are a finite number of compositions.

All elementary functions can be differentiated with derivative rules.

All elementary functions have elementary derivatives.
Binomial Expansion \((a + b)^n\)

\((a + b)^0 = 1\)

\((a + b)^1 = a + b\)

\((a + b)^2 = a^2 + 2ab + b^2\)

\((a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3\)

Pascal's Triangle

<table>
<thead>
<tr>
<th>n</th>
<th>coeff.</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>1 1</td>
</tr>
<tr>
<td>2</td>
<td>1 2 1</td>
</tr>
<tr>
<td>3</td>
<td>1 3 3 1</td>
</tr>
<tr>
<td>4</td>
<td>1 4 6 4 1</td>
</tr>
</tbody>
</table>

1st coeff.: always 1
2nd coeff.: always n
Power Rule for integer $n$, $x^n$

Let $f(x) = x^n$

$$f'(x) = \frac{d}{dx}[f] = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{1}{\Delta x} \left[ (x+\Delta x)^n - x^n \right]$$

$$= \lim_{\Delta x \to 0} \frac{1}{\Delta x} \left[ x^n + n\Delta x x^{n-1} + \ldots \Delta x^n - x^n \right]$$

$$= \lim_{\Delta x \to 0} \frac{1}{\Delta x} \left[ n \cdot \Delta x x^{n-1} + \ldots \Delta x^n \right]$$

$$= \lim_{\Delta x \to 0} \frac{n x^{n-1} + a \Delta x x^{n-2} \ldots \Delta x^{n-1}}{\Delta x}$$

$$= n x^{n-1}$$