Calculus BC Review Topics

Limits

- $\delta \epsilon$ Definition of the Limit
- ► Limit Non-Existence Three specific reasons a limit does not exist (p.51)
- ► Def. Continuity (p.70)
- ► Limits at infinity
 - Def. Horizontal Asymptote (p.199)
 - Divide by infinity \rightarrow zero (p.199-200)
 - Rational Function end behavior (p.201)
- ► Evaluating Limits
 - Numeric Method (p.48)
 - Composite limits and layering (p.61)
 - Cancellation by factoring (p.62-63)
 - Squeeze Theorem (p.65)
 - L'Hopital's Rule (p.568)
- ► Indeterminate limit forms there are *seven* (p.573).

All Indeterminate forms are convertible to
$$\frac{0}{0}$$
 or $\frac{\infty}{\infty}$.

- Converting $0 \cdot \infty$ form using $x = \frac{1}{x^{-1}}$
- Converting exponential forms using $e^{\ln x}$ method
- Converting $\infty \infty$ form using common denominator

Differential Calculus

- ► Definition of the Derivative
- ► Left and Right hand Derivatives (remember a derivative is a limit)
- ► Differentiability there are *three* conditions which all must be true. This can also be used as a condition to constrain piecewise functions.
- ► Derivative Rules (p.376)
- ► Derivations of Derivative Rules
- $\cdot \frac{d}{dx} \left[\frac{f(x)}{g(x)} \right]$ using product and chain rule $\cdot \frac{d}{dx} \left[a^{x} \right]$ using $x = e^{\ln x}$ and chain rule $\cdot \frac{d}{dx} \left[\log_{a} x \right]$ using change of base $\cdot \frac{d}{dx} \left[\log_{a} x \right]$ using $|x| = \sqrt{x^{2}}$ and chain rule

$$\cdot \frac{d}{dx} [\sin^{-1} x], \frac{d}{dx} [\tan^{-1} x]_{\text{and}} \frac{d}{dx} [\sec^{-1} x]_{\text{using}}$$

implicit differentiation and a right triangle diagram.

Differential Calculus (continued)

- ► Derivatives of inverse functions
- Implicit Differentiation
- ► Related Rates
- ► Mean Value Theorem (note conditions)
- ► Interpreting first and second derivatives
 - · Extrema, first and second derivative tests.
 - Finding tangent and normal lines
 - Position Velocity and Acceleration definitions

Integral Calculus

- ► Numeric Methods for Accumulation
 - Riemann Sums
 - Trapezoid Sums
- Def. Definite Integral (note integral def. is also a limit)
 Def. of average value of a function
- ► Indefinite Integral (anti-derivative) vs Definite Integral (limit of Riemann sum)
- ► Fundamental Theorem of Calculus
 - Evaluating derivative of an integral with functional limits
- ► Integral Power Rule
- ► Differential Substitution
- ► Logarithmic Integration

$$\int \tan x \, dx$$
, $\int \cot x \, dx$

$$\int \sec x \, dx \quad , \quad \int \csc x \, dx$$

- ► Integrals producing Inverse Trigonometric Functions
- ► Area between curves / Area of cross sections
- ► Arc Length Integrals
- ► Extrusion Integrals for Volume
 - Washer Method for revolved volumes
 - Similar Cross Section Extrusions (triangle, rectangle, semicircle etc.)
- ► Integration By Parts
 - \cdot basic use and derivation from product rule
 - · selection strategy for substitution pair (u, dv)
 - integrals of transcendental inverses: $\int \ln(x) dx$

 $\int \arcsin x \, dx$, $\int \arctan x \, dx$, $\int \operatorname{arcsec} x \, dx$

- repeated applications
- reemergence integrals

- **Calculus BC Review Topics** ► L'Hopital's Rule (p.568) ▶ Indeterminate limit forms - there are *seven* (p.573). • All Indeterminate forms are convertible to $\frac{0}{0}$ or $\frac{\infty}{\infty}$. - Converting $0 \cdot \infty$ form using $x = \frac{1}{x^{-1}}$ - Converting exponential forms using $e^{\ln x}$ method - Converting $\infty - \infty$ form using common denominator ► Arc Length Integrals ► Integration By Parts · basic use and derivation from product rule • selection strategy for substitution pair (u, dv) • integrals of transcendental inverses: $\ln(x) dx$ $\int \arcsin x \, dx$, $\int \arctan x \, dx$, $\int \operatorname{arcsec} x \, dx$ repeated applications ▶ Partial Fractions Decomposition non-repeated linear factors ► Trigonometric Integrands • use of pythagorean identities to transform even powers of cosine into sine and tangent into secant etc. power reducing identities • $\sin^n x \cdot \cos^m x$ - either m or n odd - m and n both even (power reducing \rightarrow substitution) • $\tan^n x \cdot \sec^m x$ - m even - n odd - m odd and n even ► Trigonometric Substitution • $a^2 - u^2$ in integrand \rightarrow let $\sin \theta = \frac{u}{a}$
 - $a^2 + u^2$ in integrand \rightarrow let $\tan \theta = \frac{u}{a}$ • $u^2 - a^2$ in integrand \rightarrow let $\sec \theta = \frac{u}{a}$
 - Use of a right triangle diagram to reverse substitution
 - · Conversion of limits of integration to θ using unit circle for definite integrals

- Improper Integrals
 - definition including use of limits; definition of convergence and divergence
 - Infinite upper or lower limits
 - upper or lower limits at vertical asymptote

Differential Equations

- ► Verifying Solutions
- ► Constraining General Solutions to Particular Solutions
- ► Slope Fields
- ► Euler's Method
- ► Separation of Variables
- ► Specific Systems Solutions
 - · Exponential Growth and Decay with Offset
 - Logistic Equation

Parametric Equations

- ► Parameterizing and Eliminating the Parameter
- Derivatives

$$\cdot \frac{dx}{dt}, \frac{dy}{dt}$$
 (velocity)
 $\cdot \frac{dy}{dt}$ (tangent slope)

$$\frac{1}{dx}$$
 (tangent slope)

$$\cdot \frac{d y}{dx^2}$$
 (concavity)

•
$$\frac{d^2 x}{dt^2}$$
, $\frac{d^2 y}{dt^2}$ (acceleration)

- ► Tangent lines
- ► Horizontal and vertical tangents
- ► Parametric Arc Length

Calculus BC Review Topics

Polar Coordinates

- ► Definition and Coordinate Conversions
- ► Reading Polar Graphs
- ▶ Parameterizing $r(\theta)$
 - $x=r\cos(\theta)$, $y=r\sin(\theta)$
- ► Derivatives
 - $\cdot \frac{dx}{d\theta}, \frac{dy}{d\theta}$
 - $\frac{dy}{dx}$

 - $\cdot \frac{dr}{d\theta}$
- ► Tangent Lines
 - General
 - · Horizontal and Vertical
 - Tangents through the pole
- ► Polar Area Integrals
- ▶ Polar Curve Intersections and Polar Integral Limits

Series

- ► Sequences
 - Definition
 - Limits
 - · Convergence Properties (e.g. Bounded Monotonic Converge)
 - · Factorials Definition and Simplification Operations
- ► Series Fundamentals
 - · Definition of a Series
 - Series Sum Definition
 - Convergence Tests
 - Geometric (know sum formulas)
 - P-series

- Direct Comparison
- Limit Comparison
- Integral Test
- Alt. Series Test
- Ratio Test
- Telescoping
- nth Term Test
- Absolute and Conditional Convergence
- ► Power Series
 - Taylor and Maclaurin Series Definitions
 - Forming a Taylor series by Definition
 - Taylor and Maclaurin Polynomial Definitions
 - · Power Series Convergence Properties
 - Center and Radius of Convergence
 - Interval of Convergence (endpoint convergence)
 - Derivatives and Integrals have equal convergence radius
 - Power Series Operations
 - Adding, subtracting multiplying dividing two power series
 - Differentiating and Integrating power series
 - Forming power series by variation including function composition, calculus and arithmetic operations
 - Essential Maclaurin Series
 - $-e^{x}$
 - $-\sin(x)$
 - $-\cos(x)$
 - $-\frac{1}{1-x}$