Binomial Power Series

$$(1+x)^k = 1 + kx + \frac{k(k-1)x^2}{2!} + \frac{k(k-1)(k-2)x^3}{3!} + \cdots$$

9.10.57

$$f(x) = (1+x^3)^{1/2} = 1 + \tfrac{1}{2}x^3 + \tfrac{1}{2}\cdot\tfrac{-1}{2}\cdot\frac{(x^3)^2}{2!} + \tfrac{1}{2}\cdot\tfrac{-1}{2}\cdot\tfrac{-3}{2}\cdot\frac{(x^3)^3}{3!} + \cdots$$

Binomial Series, $k = 1/2$ "x" $= x^3$

$$= 1 + \tfrac{1}{2}x^3 - \frac{1}{2^2}\cdot\frac{x^6}{2!} + \frac{3}{2^3}\cdot\frac{x^9}{3!} - \frac{1\cdot 3\cdot 5}{2^4}\,\frac{x^{12}}{4!} + \cdots$$

$$\int f(x)\,dx = F(x) = C + x + \frac{x^4}{2\cdot 4} - \frac{x^7}{2^2\, 2!\, 7} + \frac{3\, x^{10}}{2^3\cdot 3!\, 10} \cdots$$

$$\int_{.1}^{.3} f(x)\,dx = F(.3) - F(.1)$$

note: $\frac{(.3)^7}{2^2\cdot 2!\cdot 7} < \frac{1}{10{,}000}$, Alt. Series Remainder Thm. shows prev. x^4 term is sufficient

$$\int_{.1}^{.3} f(x)\,dx \approx .3 + \frac{(.3)^4}{2\cdot 4} - .1 - \frac{(.1)^4}{2\cdot 4}$$

Taylor's Theorem

Let a function be approximated by an nth degree Taylor Polynomial, $P_n(x)$.

$$f(x) \approx f(c) + \frac{f'(c)}{1!}(x-c)^1 + \cdots + \frac{f^{(n)}(c)}{n!} \cdot x^n = \underbrace{P_n(x)}$$

nth degree Taylor Poly centered at c

$$f(x) \approx P_n(x) \quad , \quad f(x) = P_n(x) + R_n(x)$$

$R_n(x)$ is the nth remainder, and gives the size of error at $x=a$ by $|R_n(a)| =$ error

$$|R_n(x)| \leq \frac{|x-c|^{n+1}}{(n+1)!} \cdot Z$$

where Z is the maximum of $f^{(n+1)}(x)$ from $x=c$ to $x=a$ where a is approximation value.

$$Z = \max |f^{(n+1)}(z)|$$

9.7.45 Use Taylor's Thm. to estimate the accuracy of the approx.

$$\cos(.3) \approx 1 - \frac{(.3)^2}{2!} + \frac{(.3)^4}{4!} = P_4(.3)$$

Taylor's Inequality

$$|R_n| \leq \frac{|x-c|^{n+1}}{(n+1)!} \underbrace{\left|f^{(n+1)}_{max}(z)\right|}_{Z}$$

$$c = 0, \quad n = 4, \quad \frac{|x-c|^{n+1}}{n+1!} = \frac{x^5}{5!} = \frac{(.3)^5}{5!}$$

$$f^{(n+1)}(x) = f^{(5)}(x) = \cos(x)$$

Z is max from $f^{(5)}(0)$ to $f^{(5)}(.3)$

$$|R_n(.3)| \leq \frac{(.3)^5}{5!} \cdot \underbrace{1}_{Z}$$