Alternating Series

- A Series is an "Alternating Series" only if every successive term changes signs

Let $\sum b_n = \sum (-1)^n a_n$ where $a_n > 0$

if $\lim_{n \to \infty} a_n = 0$ and $\underbrace{a_{n+1} \leq a_n}_{\text{Monotonic Decreasing}}$

then $\sum b_n$ converges

Alternating Series Remainder Theorem

if $\sum a_n$ converges by Alt. Series Test. Then...

$$\text{Error} = |R_N| = |S - S_N| \leq |a_N + 1|$$

where $S_N = \sum_{n=1}^{N} a_n$ is the Nth partial sum.

Absolute Convergence

IF $\sum |a_n|$ converges the $\sum a_n$ also converges

And $\sum a_n$ is Absolutely Convergent

A Series is Conditionally Convergent if $\sum a_n$ converges but $\sum |a_n|$ diverges

Ex

$$\sum a_n = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} \cdots$$ diverges (harmonic)

$$\sum b_n = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} \cdots$$ converges (alt. ser. tst)

$$= \sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n}, \quad \lim_{n \to \infty} \frac{1}{n} = 0, \quad \frac{1}{n+1} \leq \frac{1}{n} \text{ for } n > 1$$

$$\sum a_n = \sum_{n=2}^{\infty} \frac{\ln n}{n}$$ (div. direct comp. w/ $\sum \frac{1}{n}$)

$$\sum b_n = \sum_{n=2}^{\infty} (-1)^{n} \frac{\ln n}{n}, \quad \lim_{n \to \infty} \frac{\ln n}{n} = 0$$