

9.5.29 Let $a_n = \sum_{k=0}^n (-1)^{n+k} \frac{n!}{1 \cdot 3 \cdot 5 \cdots (2n-1)}$

$$= 1 - \frac{1 \cdot 2}{1 \cdot 3} + \frac{1 \cdot 2 \cdot 3}{1 \cdot 3 \cdot 5} - \frac{1 \cdot 2 \cdot 3 \cdot 4}{1 \cdot 3 \cdot 5 \cdot 7} + \dots$$

$$\frac{1}{1 \cdot 3 \cdot 5 \cdot 7 \cdots} = \frac{2 \cdot 4 \cdot 6 \cdot 8}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7 \cdot 8} = \frac{2^n (1 \cdot 2 \cdot 3 \cdot 4)}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7 \cdot 8}$$

$$= \frac{\cancel{2 \cdot n!}}{\cancel{(2n-1)!}} = g(n) = \frac{2^n n!}{(2n)!} \quad \checkmark$$

⊗ ~~$g(1) = \frac{2 \cdot 1!}{1!} = 2$~~

$$g(1) = \frac{2(1!)}{2!} = 1, \quad g(2) = \frac{2^2(2!)}{4!} = \frac{2^2(1 \cdot 2)^x}{1 \cdot 2^x \cdot 3 \cdot 4^x} = \frac{1}{1 \cdot 3}$$

$$g(3) = \frac{2^3(3!)}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6} = \frac{1}{1 \cdot 3 \cdot 5}$$

$$\sum_{n=1}^{\infty} (-1)^n a_n = \sum_{n=1}^{\infty} (-1)^n \frac{n! \cdot n! \cdot 2^n}{(2n)!}$$

⋮

This is not productive.



New approach...

9.5.29 Let $\sum (-1)^n a_n = \frac{1}{1} - \frac{1 \cdot 2}{1 \cdot 3} + \frac{1 \cdot 2 \cdot 3}{1 \cdot 3 \cdot 5} - \dots$

$a_{n+1} \leq a_n$ for $n \geq 1$: (✓)

$a_1 = 1$
 $a_2 < 1 = 1 \cdot \frac{2}{3}$
 $a_3 < a_2 = 1 \cdot \frac{2}{3} \cdot \frac{3}{5}$

(Annotations: ≤ 1 with arrows pointing to a_1 and a_2 ; $\lim_{n \rightarrow \infty} = \frac{1}{2}$ with an arrow pointing to the sequence; a_2 in a red bracket; ≤ 1 with an arrow pointing to the fraction $\frac{3}{5}$)

$a_n \leq a_{n+1} \Rightarrow a_{n+1} = a_n \cdot \frac{n+1}{2(n+1)-1} = a_n \cdot \frac{n+1}{2n+1}$

$\lim_{n \rightarrow \infty} a_n = 0$ (✓); $\lim_{n \rightarrow \infty} a_{n+1} = \lim_{n \rightarrow \infty} a_n \cdot \frac{n+1}{2n+1}$

9.5.31 Let $\sum (-1)^n a_n = \sum_{n=1}^{\infty} (-1)^{n+1} \frac{2}{e^n - e^{-n}}$

(✓) for $n \geq 1$

$a_{n+1} \leq a_n \rightarrow \frac{2}{e^{n+1} - e^{-n-1}} \leq \frac{2}{e^n - e^{-n}}$

$\frac{e \cdot e^n}{e^{n+1} - e^{-n-1}} \geq \frac{e^{-n} \cdot \frac{1}{e}}{e^n - e^{-n}}$

$e^n(e+1) \geq e^{-n}(\frac{1}{e}-1)$ true for all n

(Annotations: e^n and $(e+1)$ are both labeled "pos."; e^{-n} is labeled "pos."; $(\frac{1}{e}-1)$ is labeled "negative")

9.5.31 cont.

$$\begin{aligned} \textcircled{\checkmark} \lim_{n \rightarrow \infty} a_n &= \lim_{n \rightarrow \infty} 2 \frac{1}{e^n - e^{-n}} = \lim_{n \rightarrow \infty} \frac{1}{e^n - \frac{1}{e^n}} \\ &= \frac{1}{\infty} = 0. \quad \sum (-1)^n a_n \text{ conv. by alt. series test.} \end{aligned}$$

$$\underline{9.5.39} \quad \sum (-1)^n a_n = \sum_{n=0}^{\infty} (-1)^n \frac{1}{n!}$$

conv. by alt. series test ($a_{n+1} \leq a_n$, $\lim_{n \rightarrow \infty} a_n = 0$)

$$\text{Error} = |S - S_N| = |a_{N+1}| \leq \frac{1}{1000}$$

$$\frac{1}{(N+1)!} \leq \frac{1}{1000}, \quad (N+1)! \geq 1000$$

for $N \geq 6$

$$\frac{1}{e} \approx 1 - 1 + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5!} + \frac{1}{6!} \quad [\text{within } .001]$$

Ratio Test

$$\text{Let } L = \lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right|$$

if $L < 1$, $\sum a_n$ converges absolutely

if $L = 1$, no conclusion, use a different test

if $L > 1$ (inc. ∞) $\sum a_n$ diverges

$$\text{Ex } \sum_{n=0}^{\infty} \frac{2^n}{n!}, \quad L = \lim_{n \rightarrow \infty} \left| \frac{2^{n+1}}{(n+1)!} \cdot \frac{n!}{2^n} \right|$$

$$L = \lim_{n \rightarrow \infty} \frac{2^{n+1}}{2^n} \cdot \frac{n!}{(n+1)!} = \lim_{n \rightarrow \infty} 2 \cdot \frac{1}{n+1} = 0 < 1$$

converges absolutely by ratio test.