9.5.29 Let $\sum (-1)^{n+1} a_n = \sum_{n=1}^{\infty} (-1)^{n+1} \frac{n!}{1\cdot 3\cdot 5 \cdots (2n-1)}$

$= 1 - \frac{1\cdot 2}{1\cdot 3} + \frac{1\cdot 2\cdot 3}{1\cdot 3\cdot 5} + \frac{1\cdot 2\cdot 3\cdot 4}{1\cdot 3\cdot 5\cdot 7} \cdots$

$$\frac{1}{1\cdot 3\cdot 5\cdot 7 \cdots} = \frac{2\cdot 4\cdot 6\cdot 8}{1\cdot 2\cdot 3\cdot 4\cdot 5\cdot 6\cdot 7\cdot 8} = \frac{2^n(1\cdot 2\cdot 3\cdot 4)}{1\cdot 2\cdot 3\cdot 4\cdot 5\cdot 6\cdot 7\cdot 8}$$

$$= \cancel{\frac{2\cdot n!}{(2n-1)!}} = g(n) = \frac{2^n \cdot n!}{(2n)!}$$ ✓

⊗ ~~$g(1) = \frac{2\cdot 1!}{1!} = 2$~~

$$g(1) = \frac{2(1!)}{2!} = 1, \quad g(2) = \frac{2^1(2!)}{4!} = \frac{2^2(1\cdot 2)^{\times}}{1\cdot 2^{\times}\cdot 3\cdot 4^{\times}} = \frac{1}{1\cdot 3}$$

$$g(3) = \frac{2^3\cdot(3!)}{1\cdot 2\cdot 3\cdot 4\cdot 5\cdot 6} = \frac{1}{1\cdot 3\cdot 5}$$

This is not productive.

☹

$$\sum (-1)^n a_n = \sum_{n=1}^{\infty} (-1)^{n+1} \frac{n!\cdot n!\cdot 2^n}{(2n)!}$$

⋮

New approach...

9.5.29 Let $\sum(-1)^n a_n = \frac{1}{1} - \frac{1\cdot 2}{1\cdot 3} + \frac{1\cdot 2\cdot 3}{1\cdot 3\cdot 5} - \cdots$

$a_{n+1} \le a_n$ for $n \ge 1$: ✓

$$a_1 = 1$$

$$a_2 < 1 = 1 \cdot \overset{\le 1}{\frac{2}{3}}$$

$$a_3 < a_2 = \underbrace{1 \cdot \frac{2}{3}}_{a_2} \cdot \underset{\le 1}{\overset{\lim_{n\to\infty} = \frac{1}{2}}{\frac{3}{5}}}$$

$$a_n \le a_{n+1} \Rightarrow a_{n+1} = a_n \cdot \frac{n+1}{2(n+1)-1} = a_n \cdot \frac{n+1}{2n+1}$$

$\lim_{n\to\infty} a_n = 0$ ✓ ; $\lim_{n\to\infty} a_{n+1} = \lim_{n\to\infty} a_n \cdot \frac{n+1}{2n+1}$

9.5.31 Let $\sum(-1)^n a_n = \sum_{n=1}^{\infty} (-1)^{n+1} \frac{2}{e^n - e^{-n}}$

✓ for $n \ge 1$

$$a_{n+1} \le a_n \longrightarrow \frac{2}{e^{n+1} - e^{-n-1}} \le \frac{2}{e^n - e^{-n}}$$

$$\overset{e\cdot e^n}{e^{n+1}} - \overset{e^{-n}\cdot\frac{1}{e}}{e^{-n-1}} \ge e^n - e^{-n}$$

$$\underbrace{e^n}_{\text{pos.}}\underbrace{(e-1)}_{\text{Pos.}} \ge \underbrace{e^{-n}}_{\text{pos.}}\underbrace{\left(\frac{1}{e} - 1\right)}_{\text{negative}}$$ true for all n

9.5.31 cont.

✓ $\lim_{n\to\infty} a_n = \lim_{n\to\infty} 2\frac{1}{e^n - e^{-n}} = \lim_{n\to\infty} \frac{1}{e^n - \underbrace{\frac{1}{e^n}}_{0}}$

$= \frac{1}{\infty} = 0.$ $\sum(-1)^n a_n$ conv. by alt. series test.

9.5.39 $\sum(-1)^n a_n = \sum_{n=0}^{\infty}(-1)^n \frac{1}{n!}$

conv. by alt. series test ($a_{n+1} \leq a_n$, $\lim_{n\to\infty} a_n = 0$)

$$\text{Error} = |S - S_N| = \underbrace{|a_{N+1}| \leq \frac{1}{1000}}$$

$$\frac{1}{(N+1)!} \leq \frac{1}{1000}, \quad (N+1)! \geq 1000 \ \boxed{\text{for } N \geq 6}$$

$\frac{1}{e} \approx 1 - 1 + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \frac{1}{5!} + \frac{1}{6!}$ [within .001]

Ratio Test

Let $L = \lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right|$

if $L < 1$, $\sum a_n$ converges absolutely

if $L = 1$, no conclusion, use a different test

if $L > 1$ (ing. ∞) $\sum a_n$ diverges

Ex $\sum_{n=0}^{\infty} \frac{2^n}{n!}$, $L = \lim_{n \to \infty} \left| \frac{2^{n+1}}{(n+1)!} \cdot \frac{n!}{2^n} \right|$

$$L = \lim_{n \to \infty} \frac{2^{n+1}}{2^n} \cdot \frac{n!}{(n+1)!} = \lim_{n \to \infty} 2 \cdot \frac{1}{n+1} = 0 < 1$$

converges absolutely by ratio test.