AP Calculus B/C – Power Series AP Test Practice

FRQ.1

$$f(x) = \begin{cases} \dfrac{\cos x - 1}{x^2} & \text{for } x \neq 0 \\ -\dfrac{1}{2} & \text{for } x = 0 \end{cases}$$

The function f, defined above, has derivatives of all orders. Let g be the function defined by $g(x) = 1 + \int_0^x f(t)\,dt.$

(a) Write the first three nonzero terms and the general term of the Taylor series for $\cos x$ about $x = 0$. Use this series to write the first three nonzero terms and the general term of the Taylor series for f about $x = 0$.

(b) Use the Taylor series for f about $x = 0$ found in part (a) to determine whether f has a relative maximum, relative minimum, or neither at $x = 0$. Give a reason for your answer.

(c) Write the fifth-degree Taylor polynomial for g about $x = 0$.

(d) The Taylor series for g about $x = 0$, evaluated at $x = 1$, is an alternating series with individual terms that decrease in absolute value to 0. Use the third-degree Taylor polynomial for g about $x = 0$ to estimate the value of $g(1)$. Explain why this estimate differs from the actual value of $g(1)$ by less than $\frac{1}{6!}$.

FRQ.2

The function f has a Taylor series about $x = 1$ that converges to $f(x)$ for all x in the interval of convergence. It is known that $f(1) = 1$, $f'(1) = -\frac{1}{2}$, and the nth derivative of f at $x = 1$ is given by $f^{(n)}(1) = (-1)^n \frac{(n-1)!}{2^n}$ for $n \geq 2$.

(a) Write the first four nonzero terms and the general term of the Taylor series for f about $x = 1$.

(b) The Taylor series for f about $x = 1$ has a radius of convergence of 2. Find the interval of convergence. Show the work that leads to your answer.

(c) The Taylor series for f about $x = 1$ can be used to represent $f(1.2)$ as an alternating series. Use the first three nonzero terms of the alternating series to approximate $f(1.2)$.

(d) Show that the approximation found in part (c) is within 0.001 of the exact value of $f(1.2)$.

MC.1

To what number does the series $\sum_{k=0}^{\infty}\left(\frac{-e}{\pi}\right)^k$ converge?

(A) 0 (B) $\frac{-e}{\pi+e}$ (C) $\frac{\pi}{\pi+e}$ (D) The series does not converge.

MC.2

Which of the following is the interval of convergence for the series $\sum_{n=0}^{\infty}\frac{(x+2)^n}{2^n}$?

(A) $-4 < x < 0$

(B) $-4 \le x < 0$

(C) $-2 < x < 2$

(D) $-2 \le x < 2$

MC.3

If $f(x) = \sum_{n=1}^{\infty}\frac{x^{2n}}{n!}$, then $f'(x) =$

(A) $\frac{x^3}{3} + \frac{x^5}{5\cdot 2!} + \frac{x^7}{7\cdot 3!} + \frac{x^9}{9\cdot 4!} + \cdots + \frac{x^{(2n+1)}}{(2n+1)n!} + \cdots$

(B) $x + \frac{3x^3}{2!} + \frac{5x^5}{3!} + \frac{7x^7}{4!} + \cdots + \frac{(2n-1)x^{(2n-1)}}{n!} + \cdots$

(C) $2 + 2x^2 + x^4 + \frac{x^6}{3} + \cdots + \frac{2x^{2(n-1)}}{(n-1)!} + \cdots$

(D) $2x + 2x^3 + x^5 + \frac{x^7}{3} + \cdots + \frac{2nx^{(2n-1)}}{n!} + \cdots$

MC.4

Which of the following is a power series expansion of $\frac{e^x + e^{-x}}{2}$?

(A) $1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + \cdots + \frac{x^{2n}}{(2n)!} + \cdots$

(B) $1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots + (-1)^n\frac{x^{2n}}{(2n)!} + \cdots$

(C) $x + \frac{x^3}{3!} + \frac{x^5}{5!} + \frac{x^7}{7!} + \cdots + \frac{x^{2n+1}}{(2n+1)!} + \cdots$

(D) $x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots + (-1)^n\frac{x^{2n+1}}{(2n+1)!} + \cdots$

MC.5

Which of the following statements about the series $\sum_{n=1}^{\infty} \frac{1}{2^n - n}$ is true?

(A) The series diverges by the nth term test.

(B) The series diverges by limit comparison to the harmonic series $\sum_{n=1}^{\infty} \frac{1}{n}$.

(C) The series converges by the nth term test.

(D) The series converges by limit comparison to the geometric series $\sum_{n=1}^{\infty} \frac{1}{2^n}$.

MC.6 (calc.)

If the infinite series $S = \sum_{n=1}^{\infty} (-1)^{n+1} \frac{2}{n}$ is approximated by $P_k = \sum_{n=1}^{k} (-1)^{n+1} \frac{2}{n}$, what is the least value of k for which the alternating series error bound guarantees that $|S - P_k| < \frac{3}{100}$?

(A) 64 (B) 66 (C) 68 (D) 70

MC.7 (calc.)

Let f be a function with $f(3) = 2$, $f'(3) = -1$, $f''(3) = 6$, and $f'''(3) = 12$. Which of the following is the third-degree Taylor polynomial for f about $x = 3$?

(A) $2 - (x-3) + 3(x-3)^2 + 2(x-3)^3$

(B) $2 - (x-3) + 3(x-3)^2 + 4(x-3)^3$

(C) $2 - (x-3) + 6(x-3)^2 + 12(x-3)^3$

(D) $2 - x + 3x^2 + 2x^3$

(E) $2 - x + 6x^2 + 12x^3$

MC.8

If $f(x) = x\sin(2x)$, which of the following is the Taylor series for f about $x = 0$?

(A) $x - \frac{x^3}{2!} + \frac{x^5}{4!} - \frac{x^7}{6!} + \cdots$

(B) $x - \frac{4x^3}{2!} + \frac{16x^5}{4!} - \frac{64x^7}{6!} + \cdots$

(C) $2x - \frac{8x^3}{3!} + \frac{32x^5}{5!} - \frac{128x^7}{7!} + \cdots$

(D) $2x^2 - \frac{2x^4}{3!} + \frac{2x^6}{5!} - \frac{2x^8}{7!} + \cdots$

(E) $2x^2 - \frac{8x^4}{3!} + \frac{32x^6}{5!} - \frac{128x^8}{7!} + \cdots$

MC.9

What is the sum of the series $1+\ln 2+\frac{(\ln 2)^2}{2!}+\cdots+\frac{(\ln 2)^n}{n!}+\cdots$?

(A) $\ln 2$

(B) $\ln(1+\ln 2)$

(C) 2

(D) e^2

(E) The series diverges.

MC.10

What are all values of x for which the series $\sum_{n=1}^{\infty}\left(\frac{2}{x^2+1}\right)^n$ converges?

(A) $-1<x<1$

(B) $x>1$ only

(C) $x\geq 1$ only

(D) $x<-1$ and $x>1$ only

(E) $x\leq -1$ and $x\geq 1$

MC.11

Which of the following series converges for all real numbers x?

(A) $\sum_{n=1}^{\infty}\frac{x^n}{n}$

(B) $\sum_{n=1}^{\infty}\frac{x^n}{n^2}$

(C) $\sum_{n=1}^{\infty}\frac{x^n}{\sqrt{n}}$

(D) $\sum_{n=1}^{\infty}\frac{e^n x^n}{n!}$

(E) $\sum_{n=1}^{\infty}\frac{n!x^n}{e^n}$

MC.12

Consider the series $\sum_{n=1}^{\infty}\frac{e^n}{n!}$. If the ratio test is applied to the series, which of the following inequalities results, implying that the series converges?

(A) $\lim_{n\to\infty}\frac{e}{n!}<1$

(B) $\lim_{n\to\infty}\frac{n!}{e}<1$

(C) $\lim_{n\to\infty}\frac{n+1}{e}<1$

(D) $\lim_{n\to\infty}\frac{e}{n+1}<1$

(E) $\lim_{n\to\infty}\frac{e}{(n+1)!}<1$