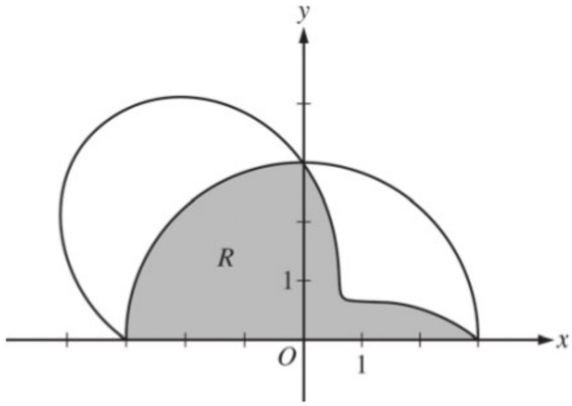
AP Calculus BC – Polar Coordinates AP Test Practice

FRQ.1 (calculator)



The graphs of the polar curves $r = 3$ and $r = 3 - 2\sin(2\theta)$ are shown in the figure above for $0 \le \theta \le \pi$.

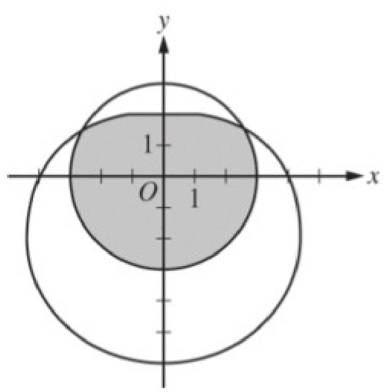
(a) Let R be the shaded region that is inside the graph of $r = 3$ and inside the graph of $r = 3 - 2\sin(2\theta)$. Find the area of R.

(b) For the curve $r = 3 - 2\sin(2\theta)$, find the value of $\frac{dx}{d\theta}$ at $\theta = \frac{\pi}{6}$.

(c) The distance between the two curves changes for $0 < \theta < \frac{\pi}{2}$. Find the rate at which the distance between the two curves is changing with respect to θ when $\theta = \frac{\pi}{3}$.

(d) A particle is moving along the curve $r = 3 - 2\sin(2\theta)$ so that $\frac{d\theta}{dt} = 3$ for all times $t \ge 0$. Find the value of $\frac{dr}{dt}$ at $\theta = \frac{\pi}{6}$.

FRQ.2 (calculator)



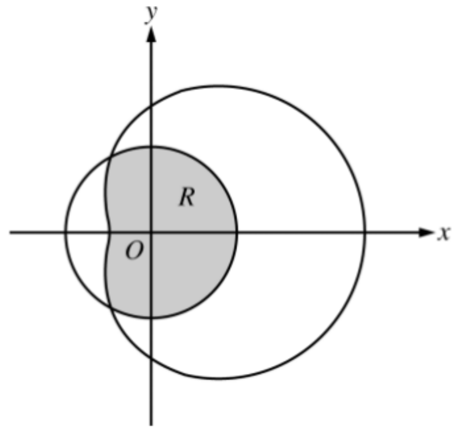
The graphs of the polar curves $r = 3$ and $r = 4 - 2\sin\theta$ are shown in the figure above. The curves intersect when $\theta = \frac{\pi}{6}$ and $\theta = \frac{5\pi}{6}$.

(a) Let S be the shaded region that is inside the graph of $r = 3$ and also inside the graph of $r = 4 - 2\sin\theta$. Find the area of S.

(b) A particle moves along the polar curve $r = 4 - 2\sin\theta$ so that at time t seconds, $\theta = t^2$. Find the time t in the interval $1 \le t \le 2$ for which the x-coordinate of the particle's position is -1.

(c) For the particle described in part (b), find the position vector in terms of t. Find the velocity vector at time $t = 1.5$.

FRQ.3 (calculator)



The graphs of the polar curves $r = 2$ and $r = 3 + 2\cos\theta$ are shown in the figure above. The curves intersect when $\theta = \frac{2\pi}{3}$ and $\theta = \frac{4\pi}{3}$.

(a) Let R be the region that is inside the graph of $r = 2$ and also inside the graph of $r = 3 + 2\cos\theta$, as shaded in the figure above. Find the area of R.

(b) A particle moving with nonzero velocity along the polar curve given by $r = 3 + 2\cos\theta$ has position $(x(t), y(t))$ at time t, with $\theta = 0$ when $t = 0$. This particle moves along the curve so that $\frac{dr}{dt} = \frac{dr}{d\theta}$. Find the value of $\frac{dr}{dt}$ at $\theta = \frac{\pi}{3}$ and interpret your answer in terms of the motion of the particle.

(c) For the particle described in part (b), $\frac{dy}{dt} = \frac{dy}{d\theta}$. Find the value of $\frac{dy}{dt}$ at $\theta = \frac{\pi}{3}$ and interpret your answer in terms of the motion of the particle.

MC.1

Which of the following integrals gives the area of the region that is bounded by the graphs of the polar equations $\theta = 0$, $\theta = \frac{\pi}{4}$, and $r = \dfrac{2}{\cos\theta + \sin\theta}$?

(A) $\displaystyle\int_0^{\pi/4} \frac{1}{\cos\theta + \sin\theta}\, d\theta$

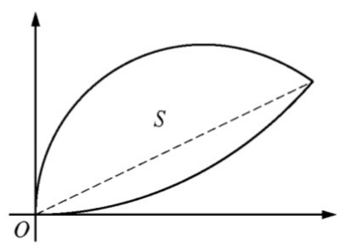
(B) $\displaystyle\int_0^{\pi/4} \frac{2}{\cos\theta + \sin\theta}\, d\theta$

(C) $\displaystyle\int_0^{\pi/4} \frac{2}{(\cos\theta + \sin\theta)^2}\, d\theta$

(D) $\displaystyle\int_0^{\pi/4} \frac{4}{(\cos\theta + \sin\theta)^2}\, d\theta$

(E) $\displaystyle\int_0^{\pi/4} \frac{2(\cos\theta - \sin\theta)^2}{(\cos\theta + \sin\theta)^4}\, d\theta$

MC.2 (calculator)



Let S be the region in the first quadrant bounded above by the graph of the polar curve $r = \cos\theta$ and bounded below by the graph of the polar curve $r = 2\theta$, as shown in the figure above. The two curves intersect when $\theta = 0.450$. What is the area of S ?

(A) 0.232 (B) 0.243 (C) 0.271 (D) 0.384

MC.3

Which of the following gives the total area enclosed by the graph of the polar curve $r = \theta\sin 2\theta$ for $0 \le \theta \le 2\pi$?

(A) $\displaystyle\int_0^{2\pi} \frac{1}{2}|\theta\sin 2\theta|\, d\theta$

(B) $\displaystyle\int_0^{2\pi} |\theta\sin 2\theta|\, d\theta$

(C) $\displaystyle\int_0^{2\pi} \frac{1}{2}(\theta\sin 2\theta)^2\, d\theta$

(D) $\displaystyle\int_0^{2\pi} (\theta\sin 2\theta)^2\, d\theta$

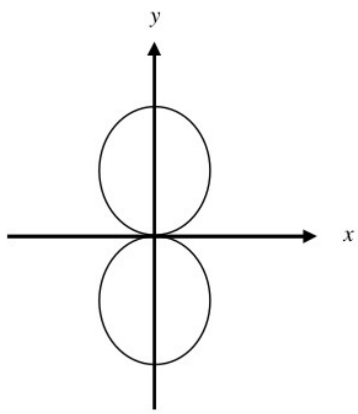
(E) $\displaystyle\int_0^{2\pi} \frac{\pi}{2}(\theta\sin 2\theta)^2\, d\theta$

MC.4

What is the slope of the line tangent to the polar curve $r = 3\theta$ at the point where $\theta = \dfrac{\pi}{2}$?

(A) $-\dfrac{\pi}{2}$ (B) $-\dfrac{2}{\pi}$ (C) 0 (D) 3

MC.5



Which of the following expressions gives the total area enclosed by the polar curve $r = \sin^2\theta$ shown in the figure above?

(A) $\dfrac{1}{2}\int_0^{\pi} \sin^2 \theta d\theta$

(B) $\int_0^{\pi} \sin^2 \theta d\theta$

(C) $\dfrac{1}{2}\int_0^{\pi} \sin^4 \theta d\theta$

(D) $\int_0^{\pi} \sin^4 \theta d\theta$

(E) $2\int_0^{\pi} \sin^4 \theta d\theta$