FRQ. 1

Consider the differential equation $\frac{dy}{dx} = 1 - y$. Let $y = f(x)$ be the particular solution to this differential equation with the initial condition $f(1) = 0$. For this particular solution, $f(x) < 1$ for all values of x.

(a) Use Euler's method, starting at $x = 1$ with two steps of equal size, to approximate $f(0)$. Show the work that leads to your answer.

(b) Find $\lim_{x \to 1} \frac{f(x)}{x^3 - 1}$. Show the work that leads to your answer.

(c) Find the particular solution $y = f(x)$ to the differential equation $\frac{dy}{dx} = 1 - y$ with the initial condition $f(1) = 0$.

FRQ.2

At the beginning of 2010, a landfill contained 1400 tons of solid waste. The increasing function W models the total amount of solid waste stored at the landfill. Planners estimate that W will satisfy the differential equation $\frac{dW}{dt} = \frac{1}{25}(W - 300)$ for the next 20 years. W is measured in tons, and t is measured in years from the start of 2010.

(a) Use the line tangent to the graph of W at $t = 0$ to approximate the amount of solid waste that the landfill contains at the end of the first 3 months of 2010 (time $t = \frac{1}{4}$).

(b) Find $\frac{d^2W}{dt^2}$ in terms of W. Use $\frac{d^2W}{dt^2}$ to determine whether your answer in part (a) is an underestimate or an overestimate of the amount of solid waste that the landfill contains at time $t = \frac{1}{4}$.

(c) Find the particular solution $W = W(t)$ to the differential equation $\frac{dW}{dt} = \frac{1}{25}(W - 300)$ with initial condition $W(0) = 1400$.

FRQ.3

Consider the differential equation $\frac{dy}{dx} = y^2(2x + 2)$. Let $y = f(x)$ be the particular solution to the differential equation with initial condition $f(0) = -1$.

(a) Find $\lim_{x \to 0} \frac{f(x) + 1}{\sin x}$. Show the work that leads to your answer.

(b) Use Euler's method, starting at $x = 0$ with two steps of equal size, to approximate $f\left(\frac{1}{2}\right)$.

(c) Find $y = f(x)$, the particular solution to the differential equation with initial condition $f(0) = -1$.

MC.1

Which of the following is the solution to the differential equation $\frac{dy}{dx} = -2xy$ with the initial condition $y(1) = 4$?

(A) $y = e^{x^2} + 4 - e$

(B) $y = e^{-x^2} + 4 - \frac{1}{e}$

(C) $y = 4e^{x^2 - 1}$

(D) $y = 4e^{-x^2 + 1}$

(E) $y = e^{-x^2 + 16}$

MC.2

If $\frac{dy}{dt} = -10e^{-t/2}$ and $y(0) = 20,$ what is the value of $y(6)$?

(A) $20e^{-6}$ (B) $20e^{-3}$ (C) $20e^{-2}$ (D) $10e^{-3}$ (E) $5e^{-3}$

MC.3

Let $y = f(x)$ be the solution to the differential equation $\frac{dy}{dx} = 2y - x$ with initial condition $f(1) = 2.$ What is the approximation for $f(0)$ obtained by using Euler's method with two steps of equal length starting at $x = 1$?

(A) $-\frac{5}{4}$ (B) -1 (C) $\frac{1}{4}$ (D) $\frac{1}{2}$ (E) $\frac{27}{4}$

MC.4

At time $t = 0$ years, a forest preserve has a population of 1500 deer. If the rate of growth of the population is modeled by $R(t) = 2000e^{0.23t}$ deer per year, what is the population at time $t = 3$?

(A) 3987 (B) 5487 (C) 8641 (D) 10,141 (E) 12,628

MC.5

Which of the following is the solution to the differential equation $\frac{dy}{dx} = e^{y+x}$ with the initial condition $y(0) = -\ln 4$?

(A) $y = -x - \ln 4$

(B) $y = x - \ln 4$

(C) $y = -\ln\left(-e^x + 5\right)$

(D) $y = -\ln\left(e^x + 3\right)$

(E) $y = \ln\left(e^x + 3\right)$

MC.6

Let $y = f(x)$ be the solution to the differential equation $\frac{dy}{dx} = 2x + y$ with initial condition $f(1) = 0$. What is the approximation for $f(2)$ obtained by using Euler's method with two steps of equal length, starting at $x = 1$?

(A) 0 (B) 1 (C) 2.75 (D) 3 (E) 6

MC.7

A population y changes at a rate modeled by the differential equation $\frac{dy}{dt} = 0.2y(1000 - y)$, where t is measured in years. What are all values of y for which the population is increasing at a decreasing rate?

(A) 500 only

(B) $0 < y < 500$ only

(C) $500 < y < 1000$ only

(D) $0 < y < 1000$

(E) $y > 1000$

MC.8

On the graph of $y = f(x)$, the slope at any point (x, y) is twice the value of x. If $f(2) = 3$, what is the value of $f(3)$?

(A) 6 (B) 7 (C) 8 (D) 9 (E) 10

MC.9

The number of moose in a national park is modeled by the function M that satisfies the logistic differential equation $\frac{dM}{dt} = 0.6M\left(1 - \frac{M}{200}\right)$, where t is the time in years and $M(0) = 50$. What is $\lim_{t \to \infty} M(t)$?

(A) 50 (B) 200 (C) 500 (D) 1000 (E) 2000

MC.10

A curve has slope $2x + 3$ at each point (x, y) on the curve. Which of the following is an equation for this cur if it passes through the point (1, 2)?

(A) $y = 5x - 3$

(B) $y = x^2 + 1$

(C) $y = x^2 + 3x$

(D) $y = x^2 + 3x - 2$

(E) $y = 2x^2 + 3x - 3$