AP Calculus BC **Test II Topic List** CEC – Wolverton

Integral Calculus Foundations

Base Integral Definitions and Theorems

-Know the following formal mathematic definitions.

Definite Integral Definition [p.273]	Indefinite Integral Definition [p.248-250]
Conditions for Integrability [p.273]	Function Average Value Definition [p.286]

Fundamental Theorem of Calculus

-[p.282] Understand that this is the logical linkage between derivatives (i.e. anti-derivatives) and integrals. Differentiation and Integration are defined as separate processes and ***proven*** logically to have inverse process properties.

-Know how to use the fundamental theorem to resolve situations involving derivatives of integrals with function limits using the general method I taught:

$$\frac{d}{dx}\left[\int_{f(x)}^{g(x)} h(t)\,dt\right] = \frac{d}{dx}\left[H(g)-H(f)\right] = h(g)\cdot g' - h(f)\cdot f'$$

-Note that using the above method does not require us to actually obtain the anti-derivative of the original integrand, H(x). [Ex: 4.4: 89, 91]

Integral Interpretation (misc.)

-The Indefinite Integral is the Anti-Derivative, and is defined as *family* of functions which can be shifted by a constant.

-Many elementary functions have non-elementary anti-derivatives.

-Definite integrals differ slightly from area bounded by a curve in that an indefinite integral can be made negative if the curve is predominately below the x-axis, or if the upper limit is less than the lower limit (i.e. integrating ‘to the right’ is the negative of integrating ‘to the left’

-Definite Integrals can also be interpreted as *the accumulation of quantity from its rate* giving the **change in that quantity** over the interval. For example the integral of *position rate (velocity)* gives the **change in position** over that time interval, not the object’s absolute position.

Definite Integral Properties

-Know properties of definite integrals including splitting of limits of integration, splitting integrals with multiple terms, function symmetry shortcuts and inequality preservation. [p.274-278]

-These properties can occasionally be used to obtain integral values without the obtaining the anti-derivative by using some other knowledge of the function. [Ex: 4.3: 41-44]

Integral Numeric Methods

-Be able to compute the following integral approximations using only a four function calculator. [Ex: 4.6:43 , n=4 only, ignore Simpson’s, calculate all using arithmetic without ‘rule’ shortcuts].

Left Hand Riemann Sum	Right Hand Riemann Sum
Midpoint Riemann Sum	Trapezoidal Sum

-Know how to use derivatives (slope and concavity) to determine over or under approximation for any of the above.

Integral Substitution Methods

Origin of Substitution

-Substitution is the integral inverse of the derivative chain rule. A change of variables is simply used to expose more clearly the anti-derivative pattern already present. [p295, 298]

Substitution and Definite Integral Limits

-Limits are tied to the variable of integration (dx, dy, du … d_) when d_ changes, limit values must change and must represent values of the new variable [p.301] [Ex: 4.5:79, Don't reverse the u-substitution before evaluating, instead change the limits]

Direct Derivative Rule Inversions

-Know all integral rules on p.383. These represent all the integrands we can find elementary anti-derivatives for with only substitution techniques (with the small exception of the hyperbolic trig. types – I will not require their use on my tests).

Exponential Anti-Derivatives

-If an exponent function's derivative appears as a factor it can be substituted.[Ex:5.5: 63,66]

-If a an exponential appears more than once it may be involved in both u and du. [Ex: 5.5:65]

Logarithmic Anti-Derivatives

-For a function with a denominator - any time a denominator's derivative can be shown to be a part of the numerator, the anti-derivative will involve a logarithm. Note that algebra such as factoring, adding zero or polynomial division may be needed to show this [Ex: 5.2: 7,17,27]

-Trigonometric functions that can be defined with sine and cosine denominators have logarithmic anti-derivatives. Know the anti-derivatives for the following. [p.337] [Ex. 5.2: 30,31]

Tangent	Cotangent
Secant	Cosecant

-For integrals with logarithms in the integrand, 1/x can be absorbed into du. [Ex. 5.2: 19,49]

Inverse Trigonometric Anti-Derivatives

-Trigonometric cofunction inverse integral formulas are not necessary because derivatives differ only by a negative sign.

-Portions of the integral formula containing $a^2 \pm u^2$ can potentially represent larger families of quadratic expressions because we can complete the square to convert to something of this form. [Ex. 5.7: 31,35]

-If an integrand needs to be 'split' , examine the derivative of your non-inverse trig. Substitution first, then make that appear. Adding zero can be used to facilitate the appearance of your non-trig-inverse differential (du). [Ex. 5.7: 33,39]

Introduction to Integral Applications

'Representative Elements' Method of Building Integral Techniques

-This is a general strategy for creating integrals to represent limit accumulation processes. This is the method used to derive the washer, extrusion, arc length and shell integral methods in your book. We will use it in the future as well. This idea is used to develop many methods in natural sciences and engineering. [p.451, 457, 476]

'Washer Method' Integrals for Revolved Volumes

$$V=\pi\int_{l_1}^{l_2} R^2-r^2\,dl$$

-Remember when visualizing this method that the process of revolving the solid and counting up its volume are separate. We are imagining the integral as counting volume by stacking cross sections along the axis, not actually revolving the shape.

-I recommend drawing the cross section then identifying the integral components one by one: Variable (dl), limits, R, r, then construct the whole integral. (Don't forget the squares and π !)

-Draw a cross section digram to help with R and r definitions using the curves defined in the problem. [Ex:7.2: 17, 21]

Extrusion Integrals for Extruded Volumes

$$V=\int_{l_1}^{l_2} A\,dl$$

-This is the more general parent method of the washer (concentric circle cross sections).

-I recommend drawing the cross section then identifying the integral components one by one: Variable, limits, A, then the whole integral.

-Remember integration direction (dl) is perpendicular to the cross section plane. [Ex: 7.2: 61]

Arc Length Integrals for Function Curves

$$D=\int_a^b \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$$

-This is derived from the pythagorean theorem or distance formula using line segment representative elements.

-Bear in mind that $\left(\frac{dy}{dx}\right)^2$ is not the second derivative, it is the **first derivative squared**.

-The radical can complicate analytic methods (finding antiderivatives). Look for ways to factor a perfect square. [Ex. 7.4: 3,5]

-Because of the above, arc length calculations often must use numeric methods.

-A function is definitely rectifiable (has finite arc length) over an interval if its derivative is continuous over an interval. You do not need to know the full conditions for a function being rectifiable.